Faraday's Law of Electromagnetic Induction

Whenever the magnetic fireld in the region of a conductor is moving, or changing in magnitude, electrons are induced to flow through the conductor.

Mutual Induction

Mutual Induction is the effect that occurs whenever a changing current in one coil induces a current in another coil near by. In fact, the two coils do not have to be coupled with an iron ring, which merely acts to strengthen an effect that would be present in any case.

The Magnitude of the Induced Electric Potential

The three factors affecting the magnitude of the induced current are:

1. the number of turns on the induction coil
2. the rate of change, or rate of motion, of the inducing magnetic field
3. the strength of the inducing magnetic field

Electromagnetic Induction

There are three distinct phenomena are involved in the process of electromagnetic induction

1. the action of the "inducing field"
2. the resulting "induced current and potential difference"
3. the magnetic field created by the induced current

http://physics.bu.edu/~duffy/PY106/Electricgenerators.html

Applications of electromagnetic induction

7-22-99

Electromagnetic induction is an incredibly useful phenomenon with a wide variety of applications. Induction is used in power generation and power transmission, and it's worth taking a look at how that's done. There are other effects with some interesting applications to consider, too, such as eddy currents.

Eddy currents

An eddy current is a swirling current set up in a conductor in response to a changing magnetic field. By Lenz¹s law, the current swirls in such a way as to create a magnetic field opposing the change; to do this in a conductor, electrons swirl in a plane perpendicular to the magnetic field.

Because of the tendency of eddy currents to oppose, eddy currents cause energy to be lost. More accurately, eddy currents transform more useful forms of energy, such as kinetic energy, into heat, which is generally much less useful. In many applications the loss of useful energy is not particularly desirable, but there are some practical applications. One is in the brakes of some trains. During braking, the metal wheels are exposed to a magnetic field from an electromagnet, generating eddy currents in the wheels. The magnetic interaction between the applied field and the eddy currents acts to slow the wheels down. The faster the wheels are spinning, the stronger the effect, meaning that as the train slows the braking force is reduced, producing a smooth stopping motion.

An electric generator

An electric motor is a device for transforming electrical energy into mechanical energy; an electric generator does the reverse, using mechanical energy to generate electricity. At the heart of both motors and generators is a wire coil in a magnetic field. In fact, the same device can be used as a motor or a generator.

When the device is used as a motor, a current is passed through the coil. The interaction of the magnetic field with the current causes the coil to spin. To use the device as a generator, the coil can be spun, inducing a current in the coil.

An AC (alternating current) generator utilizes Faraday's law of induction, spinning a coil at a constant rate in a magnetic field to induce an oscillating emf. The coil area and the magnetic field are kept constant, so, by Faraday's law, the induced emf is given by:

$$\varepsilon = -N \Delta\Phi / \Delta t = -N \Delta(BA \cos\theta) / \Delta t = -N B A \Delta(\cos\theta) / \Delta t$$

If the loop spins at a constant rate, $\theta = \omega t$. Using calculus, and taking the derivative of the cosine to get a sine (as well as bringing out a factor of $-\omega$), it's easy to show that the emf can be expressed as:

$$\varepsilon = N B A \omega \sin(\omega t)$$

The combination $N B A \omega$ represents the maximum value of the generated voltage (i.e., emf) and can be shortened to ε_o. This reduces the expression for the emf to:

$$\varepsilon = \varepsilon_o \sin(\omega t)$$

In other words, a coil of wire spun in a magnetic field at a constant rate will produce AC electricity. In North America, AC electricity from a wall socket has a frequency of 60 Hz.

A coil turning in a magnetic field can also be used to generate DC power. A DC generator uses the same kind of split-ring commutator used in a DC motor. Unlike the AC generator, the polarity of the voltage generated by a DC generator is always the same. In a very simple DC generator with a single rotating loop, the voltage level would constantly fluctuate. The voltage from many loops (out of synch with each other) is usually added together to obtain a relatively steady voltage.

Rather than using a spinning coil in a constant magnetic field, another way to utilize electromagnetic induction is to keep the coil stationary and to spin permanent magnets (providing the magnetic field and flux) around the coil. A good example of this is the way power is generated, such as at a hydro-electric power plant. The energy of falling water is used to spin permanent magnets around a fixed loop, producing AC power.

Back EMF in electric motors

You may have noticed that when something like a refrigerator or an air conditioner first turns on in your house, the lights dim momentarily. This is because of the large current required to get the motor inside these machines up to operating speed. When the motors are turning, much less current is necessary to keep them turning.

One way to analyze this is to realize that a spinning motor also acts like a generator. A motor has coils turning inside magnetic fields, and a coil turning inside a magnetic field induces an emf. This emf, known as the back emf, acts against the applied voltage that's causing the motor to spin in the first place, and reduces the current flowing through the coils. At operating speed, enough current flows to overcome any losses due to friction and to provide the necessary energy required for the motor to do work. This is generally much less current than is required to get the motor spinning in the first place.

If the applied voltage is V, then the initial current flowing through a motor with coils of resistance R is I = V / R. When the motor is spinning and generating a back emf, the current is reduced:

$I = (V - \varepsilon) / R$ $\qquad$ ε, the back emf, is usually a large fraction of V.

Mutual inductance

Faraday's law tells us that a changing magnetic flux will induce an emf in a coil. The induced emf for a coil with N loops is:

$$\varepsilon = -N \Delta\Phi / \Delta t$$

Picture two coils next to each other, end to end. If the first coil has a current going through it, a magnetic field will be produced, and a magnetic flux will pass through the second coil. Changing the current in the first coil changes the flux through the second, inducing an emf in the second coil. This is known as mutual inductance, inducing an emf in one coil by changing the current through another. The induced emf is proportional to the change in flux, which is proportional to the change in current in the first coil. The induced emf can thus be written as:

$$\varepsilon = -M \Delta I / \Delta t$$

The constant M is the mutual inductance, which depends on various factors, including the area and number of turns in coil 2, the distance between the two coils (the further apart, the less flux passes through coil 2), the relative orientation of the two coils, the number of turns / unit length in the first coil (because that's what the magnetic field produced by the first coil depends on), and whether the two coils have cores made from ferromagentic material. In other words, M is rather complicated. What's far more important in the equation above is that the emf induced in the second coil is proportional to the change in current in the first.

This effect can be put to practical use. One way to use it is in a transformer, which we'll discuss below. Another is to use it in an ammeter. Conventional ammeters are incorporated directly into circuits, but ammeters don't have to be placed in the current path for alternating current. If a loop connected to a meter is placed around a wire with an AC current in it, an emf will be induced in the loop because of the changing field from the wire, and that will produce a current in the loop, and meter, proportional to the current in the wire.

Self inductance

Coils can also induce emf's in themselves. If a changing current is passed through a coil, a changing magnetic field will be produced, inducing an emf in the coil. Again, this emf is given by:

$$\varepsilon = -N \Delta\Phi / \Delta t$$

As with mutual inductance, the induced emf is proportional to the change in current. The induced emf can be written as:

$$\varepsilon = -L \Delta I / \Delta t$$

The constant L is known as the inductance of the coil. It depends on the coil geometry, as well as on whether the coil has a core of ferromagnetic material.

We've already discussed resistors and capacitors as circuit elements. Inductors, which are simply wire coils, often with ferromagnetic cores, are another kind of circuit element. One of the main differences between these is what happens to electrical energy in them. Resistors dissipate electrical energy in the form of heat; capacitors store the energy in an electric field between the capacitor plates; and inductors store the energy in the magnetic field in the coil. The energy stored in an inductor is:

$$\text{Energy stored in an inductor : Energy} = 1/2\ LI^2$$

In general, the energy density (energy per unit volume) in a magnetic field is:

$$\text{Energy density in a magnetic field} = B^2 / (2\ \mu_0)$$

Transformers

Electricity is often generated a long way from where it is used, and is transmitted long distances through power lines. Although the resistance of a short length of power line is relatively low, over a long distance the resistance can become substantial. A power line of resistance R causes a power loss of I^2R; this is wasted as heat. By reducing the current, therefore, the I^2R losses can be minimized.

At the generating station, the power generated is given by P = VI. To reduce the current while keeping the power constant, the voltage can be increased. Using AC power, and Faraday's law of induction, there is a very simple way to increase voltage and decrease current (or vice versa), and that is to use a transformer. A transformer is made up of two coils, each with a different number of loops, linked by an iron core so the magnetic flux from one passes through the other. When the flux generated by one coil changes (as it does continually if the coil is connected to an AC power source), the flux passing through the other will change, inducing a voltage in the second coil. With AC power, the voltage induced in the second coil will also be AC.

In a standard transformer, the two coils are usually wrapped around the same iron core, ensuring that the magnetic flux is the same through both coils. The coil that provides the flux (i.e., the coil connected to the AC power source) is known as the primary coil, while the coil in which voltage is induced is known as the secondary coil. If the primary coil sets up a changing flux, the voltage in the secondary coil depends on the number of turns in the secondary:

$$V_s = -N_s \Delta\Phi / \Delta t$$

Similarly, the relationship for the primary coil is:

$$V_p = -N_p \Delta\Phi / \Delta t$$

Combining these gives the relationship between the primary and secondary voltage:

$$V_s / V_p = N_s / N_p$$

Energy (or, equivalently, power) has to be conserved, so:

$$P = V_s I_s = V_p I_p$$

So, $V_s / V_p = N_s / N_p = I_p / I_s$

If a transformer takes a high primary voltage and converts it to a low secondary voltage, the current in the secondary will be higher than that in the primary to compensate (and vice versa). A transformer in which the voltage is higher in the primary than the secondary (i.e., more turns in the primary than the secondary) is known as a step-down transformer. A transformer in which the secondary has more turns (and, therefore, higher voltage) is known as a step-up transformer.

Power companies use step-up transformers to boost the voltage to hundreds of kV before it is transmitted down a power line, reducing the current and minimizing the power lost in transmission lines. Step-down transformers are used at the other end, to decrease the voltage to the 120 or 240 V used in household circuits.

Transformers require a varying flux to work. They are therefore perfect for AC power, but do not work at all for DC power, which would keep the flux constant. The ease with which voltage and current can be transformed in an AC circuit is a large part of the reason AC power, rather than DC, is distributed by the power companies.

Although transformers dramatically reduce the energy lost to I^2R heating in power line, they don't give something for nothing. Transformers will also dissipate some energy, in the form of:

1. flux leakage - not all the magnetic flux from the primary passes through the secondary
2. self-induction - the opposition of the coils to a changing flux in them
3. heating losses in the coils of the transformer
4. eddy currents

In the iron core of a transformer, electrons would swirl in cross-sectional planes. This current would heat up the transformer, wasting power as heat. To minimize power losses due to eddy currents, the iron core is usually made up of thin laminated slices, rather than one solid piece. Current is then confined within each laminated piece, significantly reducing the swirling tendency as well as the losses by heating.

Magnetic fields and how to make them

7-14-99

Magnetism

There is a strong connection between electricity and magnetism. With electricity, there are positive and negative charges. With magnetism, there are north and south poles. Similar to charges, like magnetic poles repel each other, while unlike poles attract.

An important difference between electricity and magnetism is that in electricity it is possible to have individual positive and negative charges. In magnetism, north and south poles are always found in pairs. Single magnetic poles, known as magnetic monopoles, have been proposed theoretically, but a magnetic monopole has never been observed.

In the same way that electric charges create electric fields around them, north and south poles will set up magnetic fields around them. Again, there is a difference. While electric field lines begin on positive charges and end on negative charges, magnetic field lines are closed loops, extending from the south pole to the north pole and back again (or, equivalently, from the north pole to the south pole and back again). With a typical bar magnet, for example, the field goes from the north pole to the south pole outside the magnet, and back from south to north inside the magnet.

Electric fields come from charges. So do magnetic fields, but from moving charges, or currents, which are simply a whole bunch of moving charges. In a permanent magnet, the magnetic field comes from the motion of the electrons inside the material, or, more precisely, from something called the electron spin. The electron spin is a bit like the Earth spinning on its axis.

The magnetic field is a vector, the same way the electric field is. The electric field at a particular point is in the direction of the force a positive charge would experience if it were placed at that point. The magnetic field at a point is in the direction of the force a

north pole of a magnet would experience if it were placed there. In other words, the north pole of a compass points in the direction of the magnetic field.

One implication of this is that the magnetic south pole of the Earth is located near to the geographic North Pole. This hasn't always been the case: every once in a while (a long while) something changes inside the Earth's core, and the earth's field flips direction. Even at the present time, while the Earth's magnetic field is relatively stable, the location of the magnetic poles is slowly shifting.

The symbol for magnetic field is the letter B. The unit is the tesla (T).

The magnetic field produced by currents in wires

The simplest current we can come up with is a current flowing in a straight line, such as along a long straight wire. The magnetic field from a such current-carrying wire actually wraps around the wire in circular loops, decreasing in magnitude with increasing distance from the wire. To find the direction of the field, you can use your right hand. If you curl your fingers, and point your thumb in the direction of the current, your fingers will point in the direction of the field. The magnitude of the field at a distance r from a wire carrying a current I is given by:

Magnetic field from a long straight wire : $B = \mu_0 I / (2 \pi r)$

where μ_0 is a constant, the permeability of free space : $\mu_0 = 4\pi \times 10^{-7}$ T m / A

Currents running through wires of different shapes produce different magnetic fields. Consider a circular loop with a current traveling in a counter-clockwise direction around it (as viewed from the top). By pointing your thumb in the direction of the current, you should be able to tell that the magnetic field comes up through the loop, and then wraps around on the outside, going back down. The field at the center of a circular loop of radius r carrying a current I is given by:

Magnetic field at the center of a wire loop : $B = \mu_0 I / (2 r)$

For N loops put together to form a flat coil, the field is just multiplied by N:

Magnetic field at the center of a flat coil with N loops : $B = \mu_0 N I / (2 r)$

If a number of current-carrying loops are stacked on top of each other to form a cylinder, or, equivalently, a single wire is wound into a tight spiral, the result is known as a solenoid. The field along the axis of the solenoid has a magnitude of:

$B = \mu_0 n I$

where n = N/L is the number of turns per unit length (or, in other words, the total number of turns over the total length).

The force on a charged particle in a magnetic field

An electric field E exerts a force on a charge q. A magnetic field B will also exert a force on a charge q, but only if the charge is moving (and not moving in a direction parallel to the field). The direction of the force exerted by a magnetic field on a moving charge is perpendicular to the field, and perpendicular to the velocity (i.e., perpendicular to the direction the charge is moving).

The equation that gives the force on a charge moving at a velocity v in a magnetic field B is:

$F = q v B \sin\theta$

where θ is the angle between the magnetic field and the velocity of the charge.

This is a vector equation: F is a vector, v is a vector, and B is a vector. The only thing that is not a vector is q.

Note that when v and B are parallel (or at 180°) to each other, the force is zero. The maximum force, F = qvB, occurs when v and B are perpendicular to each other.

The direction of the force, which is perpendicular to both v and B, can be found using your right hand, applying something known as the right-hand rule. One way to do the right-hand rule is to do this: point all four fingers on your right hand in the direction of v. Then curl your fingers so the tips point in the direction of B. If you hold out your thumb as if you're hitch-hiking, your thumb will point in the direction of the force.

At least, your thumb points in the direction of the force as long as the charge is positive. A negative charge introduces a negative sign, which flips the direction of the force. So, for a negative charge your right hand lies to you, and the force on the negative charge will be opposite to the direction indicated by your right hand.

In a uniform field, a charge initially moving parallel to the field would experience no force, so it would keep traveling in straight-line motion, parallel to the field. Consider, however, a charged particle that is initially moving perpendicular to the field. This particle would experience a force perpendicular to its velocity. A force perpendicular to the velocity can only change the direction of the particle, and it can't affect the speed. In this case, the force will send the particle into uniform circular motion. The particle will travel in a circular path, with the plane of the circle being perpendicular to the direction of the field.

In this case, the force applied by the magnetic field ($F = qvB$) is the only force acting on the charged particle. Using Newton's second law gives:

$\Sigma F = q\,v\,B = m\,a$

The particle is undergoing uniform circular motion, so the acceleration is the centripetal acceleration:

$a = v^2 / r$ so, $q\,v\,B$

$= m\,v^2 / r$

A factor of v cancels out on both sides

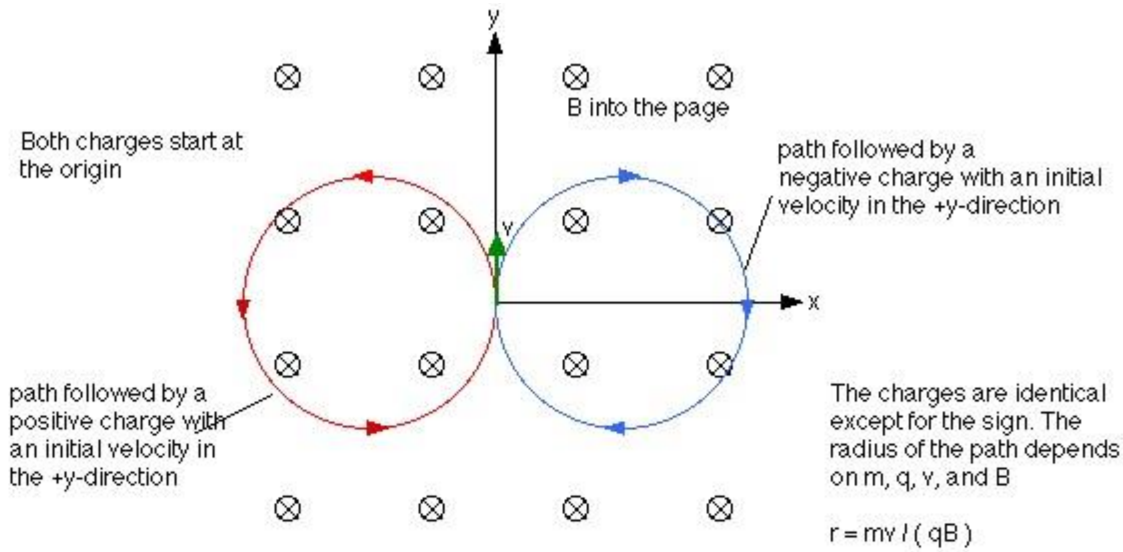
The field does not affect v-parallel in any way; this is where the straight line motion comes from. On the other hand, the field and v-perpendicular combine to produce circular motion. Superimpose the two motions and you get a spiral path.

Working in three dimensions

With the force, velocity, and field all perpendicular to each other, we have to work in three dimensions. It can be hard to draw in 3-D on a 2-D surface such as a piece of paper or a chalk board, so to represent something pointing in the third dimension, perpendicular to the page or board, we usually draw the direction as either a circle with a dot in the middle or a circle with an X in the middle.

Think of an arrow with a tip at one end and feathers at the other. If you look at an arrow coming toward you, you see the tip; if you look at an arrow going away from you, you see the X of the feathers. A circle with a dot, then, represents something coming out of the page or board at you; a circle with an X represents something going into the page or board.

The following diagram shows the path followed by two charges, one positive and one negative, in a magnetic field that points into the page:



Forces on currents in magnetic fields

7-15-99

The magnetic force on a current-carrying wire

A magnetic field will exert a force on a single moving charge, so it follows that it will also exert a force on a current, which is a collection of moving charges.

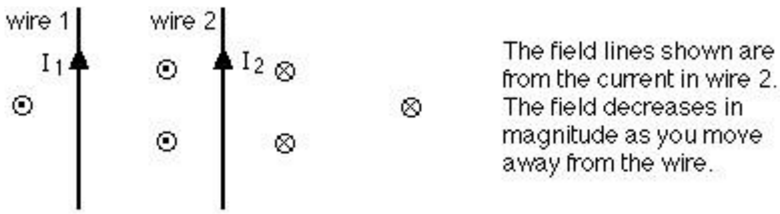
The force experienced by a wire of length l carrying a current I in a magnetic field B is given by

force on a current-carrying wire : $F = I \ell B \sin\theta$

Again, the right-hand rule can be used to find the direction of the force. In this case, your thumb points in the direction of the current, your fingers point in the direction of B. Your palm gives the direction of F.

The force between two parallel wires

Parallel wires carrying currents will exert forces on each other. One wire sets up a magnetic field that influences the other wire, and vice versa. When the current goes the same way in the two wires, the force is attractive. When the currents go opposite ways, the force is repulsive. You should be able to confirm this by looking at the magnetic field set up by one current at the location of the other wire, and by applying the right-hand rule.



Here's the approach. In the picture above, both wires carry current in the same direction. To find the force on wire 1, look first at the magnetic field produced by the current in wire 2. Everywhere to the right of wire 2, the field due to that current is into the page. Everywhere to the left, the field is out of the page. Thus, wire 1 experiences a field that is out of the page.

Now apply the right hand rule to get the direction of the force experienced by wire 1. The current is up (that's your fingers) and the field is out of the page (curl your fingers that way). Your thumb should point right, towards wire 2. The same process can be used to figure out the force on wire 2, which points toward wire 1.

Reversing one of the currents reverses the direction of the forces.

The magnitude of the force in this situation is given by F = IlB. To get the force on wire 1, the current is the current in wire 1. The field comes from the other wire, and is proportional to the current in wire 2. In other words, both currents come into play. Using the expression for the field from a long straight wire, the force is given by:

$F = \mu_0 I_1 I_2 \ell / 2 \pi L$, where L is the distance between the wires.

Note that it is often the force per unit length, F / l, that is asked for rather than the force.

The torque on a current loop

A very useful effect is the torque exerted on a loop by a magnetic field, which tends to make the loop rotate. Many motors are based on this effect.

The torque on a coil with N turns of area A carrying a current I is given by:

torque on a coil : $\tau = N I A B \sin\phi$

ϕ is the angle between the magnetic field and the normal to the plane of the coil.

The combination NIA is usually referred to as the magnetic moment of the coil. It is a vector normal (i.e., perpendicular) to the loop. If you curl your fingers in the direction of the current around the loop, your thumb will point in the direction of the magnetic moment.

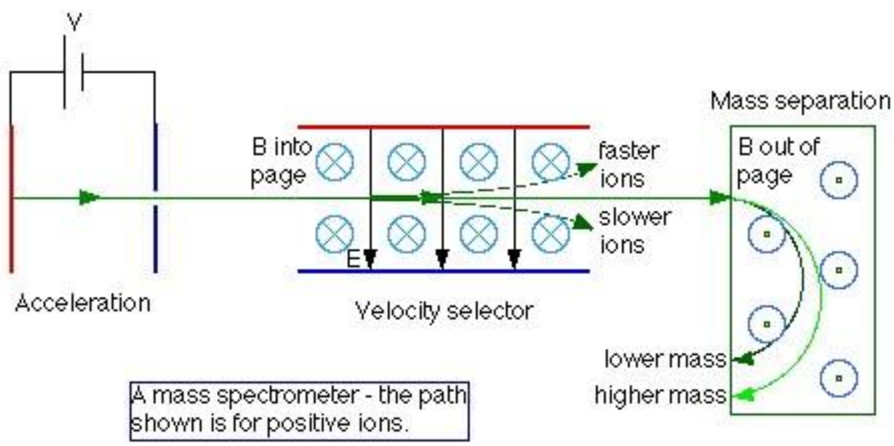
Applications of magnetic forces and fields

There are a number of good applications of the principle that a magnetic field exerts a

force on a moving charge. One of these is the mass spectrometer : a mass spectrometer separates charged particles (usually ions) based on their mass.

The mass spectrometer

The mass spectrometer involves three steps. First the ions are accelerated to a particular velocity; then just those ions going a particular velocity are passed through to the third and final stage where the separation based on mass takes place. It's worth looking at all three stages because they all rely on principles we've learned in this course.



Step 1 - Acceleration

In physics, we usually talk about charged particles (or ions) being accelerated through a potential difference of so many volts. What this means is that we're applying a voltage across a set of parallel plates, and then injecting the ions at negligible speed into the are between the plates near the plate that has the same sign charge as the ions. The ions will be repelled from that plate, attracted to the other one, and if we cut a hole in the second one they will emerge with a speed that depends on the voltage.

The simplest way to figure out how fast the ions are going is to analyze it in terms of energy. When the ions enter the region between the plates, the ions have negligible kinetic energy, but plenty of potential energy. If the plates have a potential difference of V, the potential energy is simply U = qV. When the ions reach the other plate, all this energy has been converted into kinetic energy, so the speed can be calculated from:

$1/2\, mv^2 = qV$

Step 2 - the velocity selector

The ions emerge from the acceleration stage with a range of speeds. To distinguish between the ions based on their masses, they must enter the mass separation stage with identical velocities. This is done using a velocity selector, which is designed to allow ions of only a particular velocity to pass through undeflected. Slower ions will generally be deflected one way, while faster ions will deflect another way. The velocity selector uses both an electric field and a magnetic field, with the fields at right angles to each other, as well as to the velocity of the incoming charges.

Let's say the ions are positively charged, and move from left to right across the page. An electric field pointing down the page will tend to deflect the ions down the page with a force of $F = qE$. Now, add a magnetic field pointing into the page. By the right hand rule, this gives a force of $F = qvB$ which is directed up the page. Note that the magnetic force depends on the velocity, so there will be some particular velocity where the electric force qE and the magnetic force qvB are equal and opposite. Setting the forces equal, $qE = qvB$, and solving for this velocity gives $v = E / B$. So, a charge of velocity $v = E / B$ will experience no net force, and will pass through the velocity selector undeflected.

Any charge moving slower than this will have the magnetic force reduced, and will bend in the direction of the electric force. A charge moving faster will have a larger magnetic force, and will bend in the direction of the magnetic force.

A velocity selector works just as well for negative charges, the only difference being that the forces are in the opposite direction to the way they are for positive charges.

Step 3 - mass separation

All these ions, with the same charge and velocity, enter the mass separation stage, which is simply a region with a uniform magnetic field at right angles to the velocity of the ions. Such a magnetic field causes the charges to follow circular paths of radius $r = mv / qB$. The only thing different for these particles is the mass, so the heavier ions travel in a circular path of larger radius than the lighter ones.

The particles are collected after they have traveled half a circle in the mass separator. All the particles enter the mass separator at the same point, so if a particle of mass m_1 follows a circular path of radius r_1, and a second mass m_2 follows a circular path of radius r_2, after half a circle they will be separated by the difference between the diameters of the paths after half a circle. The separation is

$s = 2\,(r_2 - r_1).$

The Hall Effect

Another good application of the force exerted by moving charges is the Hall effect. The Hall effect is very interesting, because it is one of the few physics phenomena that tell us that current in wires is made up of negative charges. It is also a common way of measuring the strength of a magnetic field.

Start by picturing a wire of square cross-section, carrying a current out of the page. We want to figure out whether the charges flowing in that wire are positive, and out of the page, or negative, flowing in to the page. There is a uniform magnetic field pointing down the page.

First assume that the current is made up of positive charges flowing out of the page. With a magnetic field down the page, the right-hand rule indicates that these positive charges experience a force to the right. This will deflect the charges to the right, piling up positive charge on the right and leaving a deficit of positive charge (i.e., a net negative charge) on the left. This looks like a set of charged parallel plates, so an electric field pointing from right to left is set up inside the wire by these charges. The field builds up until the force experienced by the charges in this electric field is equal and opposite to the force applied on the charges by the magnetic field.

With an electric field, there is a potential difference across the wire that can be measured with a voltmeter. This is known as the Hall voltage, and in the case of the positive charges, the sign on the Hall voltage would indicate that the right side of the wire is positive.

Now, what if the charges flowing through the wire are really negative, flowing into the page? Applying the right-hand rule indicates a magnetic force pointing right. This tends to pile up negative charges on the right, resulting in a deficit of negative charge (i.e., a net positive charge) on the left. As above, an electric field is the result, but this time it points from left to right. Measuring the Hall voltage this time would indicate that the left side of the wire is negative.

So, the potential difference set up across the wire is of one sign for negative charges, and the other sign for positive charges, allowing us to distinguish between the two, and to tell that when charges flow in wires, they are negative. Note that the electric field, and the Hall voltage, increases as the magnetic field increases, which is why the Hall effect can be used to measure magnetic fields.

Magnetic materials

7-20-99

Ferromagnets

A ferromagnetic material is one that has magnetic properties similar to those of iron. In other words, you can make a magnet out of it. Some other ferromagnetic materials are nickel, cobalt, and alnico, an aluminum-nickel-cobalt alloy.

Magnetic fields come from currents. This is true even in ferromagnetic materials; their magnetic properties come from the motion of electrons in the atoms. Each electron has a spin. This is a quantum mechanical phenomenon that is difficult to make a comparison to, but can be thought of as similar to the rotation of the Earth about its axis.

Electron spins are in one of two states, up or down. This is another way of stating that the magnetic quantum number can be +1/2 or -1/2. Electrons are arranged in shells and orbitals in an atom. If they fill the orbitals so that there are more spins pointing up than down (or vice versa), each atom will act like a tiny magnet.

That's not the whole picture, however; in non-magnetic materials such as aluminum, neighboring atoms do not align themselves with each other or with an external magnetic field. In ferromagnetic materials, the spins of neighboring atoms do align (through a quantum effect known as exchange coupling), resulting in small (a tenth of a millimeter, or less) neighborhoods called domains where all the spins are aligned. When a piece of unmagnetized iron (or other ferromagnetic material) is exposed to an external magnetic field, two things happen. First, the direction of magnetization (the way the spins point) of each domain will tend to shift towards the direction of the field. Secondly, domains which are aligned with the field will expand to take over regions occupied by domains aligned opposite to the field. This is what is meant by magnetizing a piece of iron.

Iron comes in two forms, hard and soft. If you were hit on the head with a soft iron bar, it would still feel very hard; soft is simply a term describing the magnetic properties. In hard iron, the domains will not shift back to their starting points when the field is taken away. In soft iron, the domains return to being randomly aligned when the field is removed.

Hard iron is used in permanent magnets. To make a permanent magnet, a piece of hard iron is placed in a magnetic field. The domains align with the field, and retain a good deal of that alignment when the field is removed, resulting in a magnet.

An electromagnet, in contrast, uses soft iron; this allows the field to be turned on and off. It's easy to make an electromagnet. One method is to coil a wire around a nail (made of iron or steel), and connect the two ends of the wire to a battery. A coil of wire with a current running through it acts as a magnet all by itself, so why is the nail necessary? The answer is that when the domains in the nail align with the field produced by the current, the magnetic field is magnified by a large factor, typically by 100 - 1000 times.

Magnetic effects are sensitive to temperature. It is much easier to keep permanent magnets magnetized at low temperatures, because at higher temperatures the atoms tend to move around much more, throwing the spins out of alignment. Above a critical temperature known as the Curie temperature, ferromagnets lose their ferromagnetic properties.

Induced EMF

7-21-99

Linking electricity and magnetism

So far we've dealt with electricity and magnetism as separate topics. From now on we'll investigate the inter-connection between the two, starting with the concept of induced EMF. This involves generating a voltage by changing the magnetic field that passes through a coil of wire.

We'll come back and investigate this quantitatively, but for now we can just play with magnets, magnetic fields, and coils of wire. You'll be doing some more playing like this in one of the labs. There are also some coils and magnets available in the undergraduate resource room - please feel free to use them.

First, connect a coil of wire to a galvanometer, which is just a very sensitive device we can use to measure current in the coil. There is no battery or power supply, so no current should flow. Now bring a magnet close to the coil. You should notice two things:

1. If the magnet is held stationary near, or even inside, the coil, no current will flow through the coil.
2. If the magnet is moved, the galvanometer needle will deflect, showing that current is flowing through the coil. When the magnet is moved one way (say, into the coil), the needle deflects one way; when the magnet is moved the other way (say, out of the coil), the needle deflects the other way. Not only can a moving magnet cause a current to flow in the coil, the direction of the current depends on how the magnet is moved.

How can this be explained? It seems like a constant magnetic field does nothing to the coil, while a changing field causes a current to flow.

To confirm this, the magnet can be replaced with a second coil, and a current can be set up in this coil by connecting it to a battery. The second coil acts just like a bar magnet. When this coil is placed next to the first one, which is still connected to the galvanometer, nothing happens when a steady current passes through the second coil. When the current in the second coil is switched on or off, or changed in any way, however, the galvanometer responds, indicating that a current is flowing in the first coil.

You also notice one more thing. If you squeeze the first coil, changing its area, while it's sitting near a stationary magnet, the galvanometer needle moves, indicating that current is flowing through the coil.

What you can conclude from all these observations is that a changing magnetic field will produce a voltage in a coil, causing a current to flow. To be completely accurate, if the magnetic flux through a coil is changed, a voltage will be produced. This voltage is known as the induced emf.

The magnetic flux is a measure of the number of magnetic field lines passing through an area. If a loop of wire with an area A is in a magnetic field B, the magnetic flux is given by:

$\Phi = B A \cos\phi$, where ϕ is the angle between the magnetic field B and the vector A, which is perpendicular to the plane of the loop.

If the flux changes, an emf will be induced. There are therefore three ways an emf can be induced in a loop:

1. Change the magnetic field
2. Change the area of the loop
3. Change the angle between the field and the loop

Faraday's law of induction

We'll move from the qualitative investigation of induced emf to the quantitative picture. As we have learned, an emf can be induced in a coil if the magnetic flux through the coil is changed. It also makes a difference how fast the change is; a quick change induces more emf than a gradual change. This is summarized in Faraday's law of induction. The induced emf in a coil of N loops produced by a change in flux in a certain time interval is given by:

Faraday's law of induction : $\varepsilon = -N \Delta\Phi / \Delta t$

Recalling that the flux through a loop of area A is given by

$\Phi = BA \cos\phi$,

Faraday's law can be written:

$\varepsilon = -N \Delta(BA \cos\phi) / \Delta t$

The negative sign in Faraday's law comes from the fact that the emf induced in the coil acts to oppose any change in the magnetic flux. This is summarized in Lenz's law.

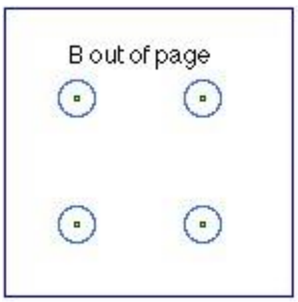
Lenz's law: The induced emf generates a current that sets up a magnetic field which acts to oppose the change in magnetic flux.

Another way of stating Lenz's law is to say that coils and loops like to maintain the status quo (i.e., they don't like change). If a coil has zero magnetic flux, when a magnet is brought close then, while the flux is changing, the coil will set up its own magnetic field that points opposite to the field from the magnet. On the other hand, a coil with a particular flux from an external magnetic field will set up its own magnetic field in an attempt to maintain the flux at a constant level if the external field (and therefore flux) is changed.

An example

Consider a flat square coil with N = 5 loops. The coil is 20 cm on each side, and has a

magnetic field of 0.3 T passing through it. The plane of the coil is perpendicular to the magnetic field: the field points out of the page.



With a constant field, there is no induced emf, and no current is induced to flow around the coil

(a) If nothing is changed, what is the induced emf?

There is only an induced emf when the magnetic flux changes, and while the change is taking place. If nothing changes, the induced emf is zero.

(b) The magnetic field is increased uniformly from 0.3 T to 0.8 T in 1.0 seconds. While the change is taking place, what is the induced emf in the coil?

Probably the most straight-forward way to approach this is to calculate the initial and final magnetic flux through the coil.

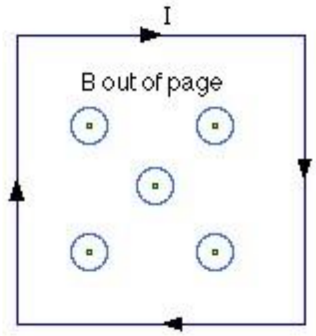
Initial magnetic flux : $\Phi_0 = B_0 A = 0.3\,(0.2)^2 = 0.012\ \mathrm{T\,m^2}$
Final magnetic flux : $\Phi_f = B_f A = 0.8\,(0.2)^2 = 0.032\ \mathrm{T\,m^2}$

The induced emf is then:

$$\varepsilon = -N\,\Delta\Phi / \Delta t = -N\,(\Phi_f - \Phi_0) / \Delta t = -5\,(0.032 - 0.012) / 1.0 = -0.1\ \mathrm{V}$$

(c) While the magnetic field is changing, the emf induced in the coil causes a current to flow. Does the current flow clockwise or counter-clockwise around the coil?

To answer this, apply Lenz's law, as well as the right-hand rule. While the magnetic field is being changed, the magnetic flux is being increased out of the page. According to Lenz's law, the emf induced in the loop by this changing flux produces a current that sets up a field opposing the change. The field set up by the current in the coil, then, points into the page, opposite to the direction of the increase in flux. To produce a field into the page, the current must flow clockwise around the loop. This can be found from the right hand rule.



While the field changes, a current flows in the loop. The direction of current in this case is clockwise, because that tends to cancel the increase in the external flux. The field produced by the flowing current is into the page.

One way to apply the rule is this. Point the thumb on your right hand in the direction of the required field, into the page in this case. If you curl your fingers, they curl in the direction the current flows around the loop - clockwise.

Motional emf

Let's say you have a metal rod, and decide to connect that to your galvanometer. If the rod is stationary in a magnetic field, nothing happens. If you move the rod through the field, however, an emf is induced between the ends of the rod causing current to flow. This is because when you move the metal rod through the field, you are moving all the electrons in the rod. These moving charges are deflected by the field toward one end of the rod, creating a potential difference. This is known as motional emf. Motional emf can even be measured on airplanes. As the plane flies through the Earth's magnetic field, an emf is induced between the wingtips.

Motional emf is largest when the direction of motion of the piece of metal is perpendicular to the rod and perpendicular to the magnetic field. When this is true, the motional emf is proportional to the speed of the rod, the length (L) of the rod, and the magnetic field:

motional emf : $\varepsilon = vBL$

If the metal rod is part of a complete circuit, the induced emf will cause a current to flow. Because it's in a magnetic field, the rod experiences a force because of the interaction between the field and the current. This force always acts to oppose the motion of the rod.

When we looked at DC motors, we saw how the force exerted on a current flowing around a coil in a magnetic field can produce rotation, transforming electrical energy to mechanical energy. Motional emf is a good example of how mechanical energy, energy associated with motion, can be transformed to electrical energy.

AC Circuits

7-23-99

Alternating current

Direct current (DC) circuits involve current flowing in one direction. In alternating current (AC) circuits, instead of a constant voltage supplied by a battery, the voltage oscillates in a sine wave pattern, varying with time as:

$$V = V_0 \sin \omega t$$

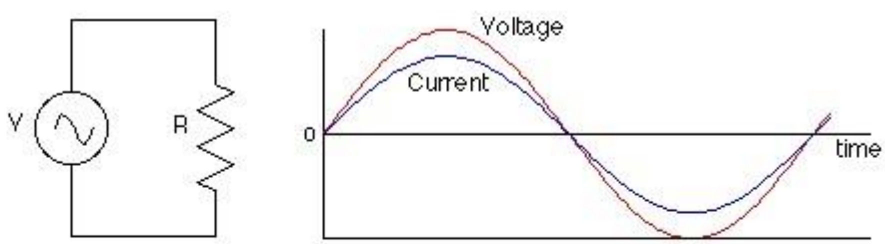
In a household circuit, the frequency is 60 Hz. The angular frequency is related to the frequency, f, by:

$$\omega = 2 \pi f$$

Vo represents the maximum voltage, which in a household circuit in North America is about 170 volts. We talk of a household voltage of 120 volts, though; this number is a kind of average value of the voltage. The particular averaging method used is something called root mean square (square the voltage to make everything positive, find the average, take the square root), or rms. Voltages and currents for AC circuits are generally expressed as rms values. For a sine wave, the relationship between the peak and the rms average is:

rms value = 0.707 peak value

Resistance in an AC circuit

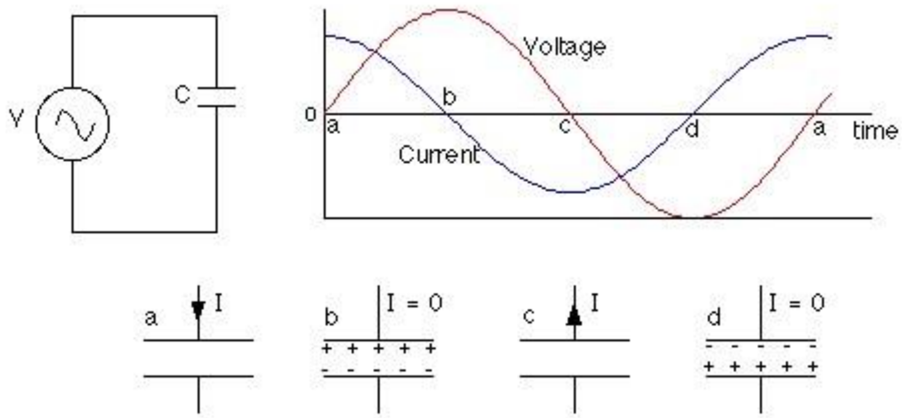


The relationship V = IR applies for resistors in an AC circuit, so

$$I = V / R = (V_0 / R) \sin(\omega t) = I_0 \sin(\omega t)$$

In AC circuits we'll talk a lot about the phase of the current relative to the voltage. In a circuit which only involves resistors, the current and voltage are in phase with each other, which means that the peak voltage is reached at the same instant as peak current. In circuits which have capacitors and inductors (coils) the phase relationships will be quite different.

Capacitance in an AC circuit



Consider now a circuit which has only a capacitor and an AC power source (such as a wall outlet). A capacitor is a device for storing charging. It turns out that there is a 90° phase difference between the current and voltage, with the current reaching its peak 90° (1/4 cycle) before the voltage reaches its peak. Put another way, the current leads the voltage by 90° in a purely capacitive circuit.

To understand why this is, we should review some of the relevant equations, including:

relationship between voltage and charge for a capacitor: $CV = Q$

relationship between current and the flow of charge : $I = \Delta Q / \Delta t$

The AC power supply produces an oscillating voltage. We should follow the circuit through one cycle of the voltage to figure out what happens to the current.

Step 1 - At point a (see diagram) the voltage is zero and the capacitor is uncharged. Initially, the voltage increases quickly. The voltage across the capacitor matches the power supply voltage, so the current is large to build up charge on the capacitor plates. The closer the voltage gets to its peak, the slower it changes, meaning less current has to flow. When the voltage reaches a peak at point b, the capacitor is fully charged and the current is momentarily zero.

Step 2 - After reaching a peak, the voltage starts dropping. The capacitor must discharge now, so the current reverses direction. When the voltage passes through zero at point c, it's changing quite rapidly; to match this voltage the current must be large and negative.

Step 3 - Between points c and d, the voltage is negative. Charge builds up again on the capacitor plates, but the polarity is opposite to what it was in step one. Again the current is negative, and as the voltage reaches its negative peak at point d the current drops to zero.

Step 4 - After point d, the voltage heads toward zero and the capacitor must discharge. When the voltage reaches zero it's gone through a full cycle so it's back to point a again to repeat the cycle.

The larger the capacitance of the capacitor, the more charge has to flow to build up a particular voltage on the plates, and the higher the current will be. The higher the frequency of the voltage, the shorter the time available to change the voltage, so the larger the current has to be. The current, then, increases as the capacitance increases and as the frequency increases.

Usually this is thought of in terms of the effective resistance of the capacitor, which is known as the capacitive reactance, measured in ohms. There is an inverse relationship between current and resistance, so the capacitive reactance is inversely proportional to the capacitance and the frequency:

A capacitor in an AC circuit exhibits a kind of resistance called capacitive reactance, measured in ohms. This depends on the frequency of the AC voltage, and is given by:

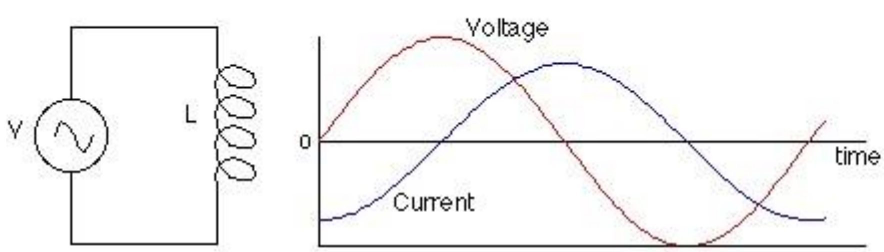
capacitive reactance: $X_c = 1 / \omega C = 1 / 2\pi f C$

We can use this like a resistance (because, really, it is a resistance) in an equation of the form V = IR to get the voltage across the capacitor:

$V = I X_c$

Note that V and I are generally the rms values of the voltage and current.

Inductance in an AC circuit



An inductor is simply a coil of wire (often wrapped around a piece of ferromagnet). If we now look at a circuit composed only of an inductor and an AC power source, we will again find that there is a 90° phase difference between the voltage and the current in the inductor. This time, however, the current lags the voltage by 90°, so it reaches its peak 1/4 cycle after the voltage peaks.

The reason for this has to do with the law of induction:

$\varepsilon = -N \Delta\Phi / \Delta t$ or $\varepsilon = -L \Delta I / \Delta t$

Applying Kirchoff's loop rule to the circuit above gives:

$V - L \Delta I / \Delta t = 0$ so $V = L \Delta I / \Delta t$

As the voltage from the power source increases from zero, the voltage on the inductor matches it. With the capacitor, the voltage came from the charge stored on the capacitor plates (or, equivalently, from the electric field between the plates). With the inductor, the voltage comes from changing the flux through the coil, or, equivalently, changing the current through the coil, which changes the magnetic field in the coil.

To produce a large positive voltage, a large increase in current is required. When the voltage passes through zero, the current should stop changing just for an instant. When the voltage is large and negative, the current should be decreasing quickly. These conditions can all be satisfied by having the current vary like a negative cosine wave, when the voltage follows a sine wave.

How does the current through the inductor depend on the frequency and the inductance? If the frequency is raised, there is less time to change the voltage. If the time interval is reduced, the change in current is also reduced, so the current is lower. The current is also reduced if the inductance is increased.

As with the capacitor, this is usually put in terms of the effective resistance of the inductor. This effective resistance is known as the inductive reactance. This is given by:

$X_L = \omega L = 2\pi f L$

where L is the inductance of the coil (this depends on the geometry of the coil and whether its got a ferromagnetic core). The unit of inductance is the henry.

As with capacitive reactance, the voltage across the inductor is given by:

$V = IX_L$

Where does the energy go?

One of the main differences between resistors, capacitors, and inductors in AC circuits is in what happens with the electrical energy. With resistors, power is simply dissipated as heat. In a capacitor, no energy is lost because the capacitor alternately stores charge and then gives it back again. In this case, energy is stored in the electric field between the capacitor plates. The amount of energy stored in a capacitor is given by:

energy in a capacitor : $\text{Energy} = 1/2\ CV^2$

In other words, there is energy associated with an electric field. In general, the energy density (energy per unit volume) in an electric field with no dielectric is:

Energy density in an electric field $= 1/2\ \varepsilon_0 E^2$

With a dielectric, the energy density is multiplied by the dielectric constant.

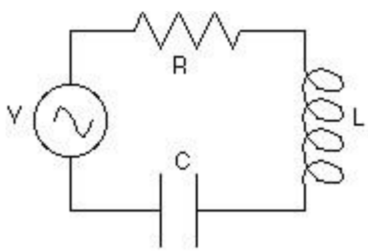
There is also no energy lost in an inductor, because energy is alternately stored in the magnetic field and then given back to the circuit. The energy stored in an inductor is:

energy in an inductor: Energy = $1/2\ LI^2$

Again, there is energy associated with the magnetic field. The energy density in a magnetic field is:

Energy density in a magnetic field = $B^2 / (2\ \mu_0)$

RLC Circuits



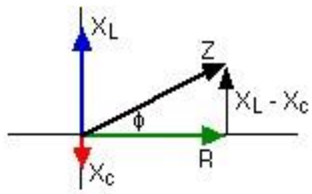
Consider what happens when resistors, capacitors, and inductors are combined in one circuit. If all three components are present, the circuit is known as an RLC circuit (or LRC). If only two components are present, it's either an RC circuit, an RL circuit, or an LC circuit.

The overall resistance to the flow of current in an RLC circuit is known as the impedance, symbolized by Z. The impedance is found by combining the resistance, the capacitive reactance, and the inductive reactance. Unlike a simple series circuit with resistors, however, where the resistances are directly added, in an RLC circuit the resistance and reactances are added as vectors.

This is because of the phase relationships. In a circuit with just a resistor, voltage and current are in phase. With only a capacitor, current is 90° ahead of the voltage, and with just an inductor the reverse is true, the voltage leads the current by 90°. When all three components are combined into one circuit, there has to be some compromise.

To figure out the overall effective resistance, as well as to determine the phase between the voltage and current, the impedance is calculated like this. The resistance R is drawn along the +x-axis of an x-y coordinate system. The inductive reactance is at 90° to this, and is drawn along the +y-axis. The capacitive reactance is also at 90° to the resistance, and is 180° different from the inductive reactance, so it's drawn along the -y-axis. The impedance, Z, is the sum of these vectors, and is given by:

$Z = [R^2 + (X_L - X_C)^2]^{1/2}$



The current and voltage in an RLC circuit are related by V = IZ. The phase relationship between the current and voltage can be found from the vector diagram: its the angle between the impedance, Z, and the resistance, R. The angle can be found from:

$\tan\phi = (X_L - X_C) / R$

If the angle is positive, the voltage leads the current by that angle. If the angle is negative, the voltage lags the currents.

The power dissipated in an RLC circuit is given by:

$P = VI \cos\phi$ $\cos\phi$ is known as the power factor in the circuit

Note that all of this power is lost in the resistor; the capacitor and inductor alternately store energy in electric and magnetic fields and then give that energy back to the circuit.

Electromagnetic waves

7-26-99

At this point in the course we'll move into optics. This might seem like a separate topic from electricity and magnetism, but optics is really a sub-topic of electricity and magnetism. This is because optics deals with the behavior of light, and light is one example of an electromagnetic wave.

Light and other electromagnetic waves

Light is not the only example of an electromagnetic wave. Other electromagnetic waves include the microwaves you use to heat up leftovers for dinner, and the radio waves that are broadcast from radio stations. An electromagnetic wave can be created by accelerating charges; moving charges back and forth will produce oscillating electric and magnetic fields, and these travel at the speed of light. It would really be more accurate to call the speed "the speed of an electromagnetic wave", because light is just one example of an electromagnetic wave.

speed of light in vacuum: $c = 3.00 \times 10^8$ m/s

As we'll go into later in the course when we get to relativity, c is the ultimate speed limit in the universe. Nothing can travel faster than light in a vacuum.

There is a wonderful connection between c, the speed of light in a vacuum, and the constants that appeared in the electricity and magnetism equations, the permittivity of free space and the permeability of free space. James Clerk Maxwell, who showed that all of electricity and magnetism could be boiled down to four basic equations, also worked out that:

$$c = 1 / [\varepsilon_0 \mu_0]^{1/2}$$

This clearly shows the link between optics, electricity, and magnetism.

Creating an electromagnetic wave

We've already learned how moving charges (currents) produce magnetic fields. A constant current produces a constant magnetic field, while a changing current produces a changing field. We can go the other way, and use a magnetic field to produce a current, as long as the magnetic field is changing. This is what induced emf is all about. A steadily-changing magnetic field can induce a constant voltage, while an oscillating magnetic field can induce an oscillating voltage.

Focus on these two facts:

1. an oscillating electric field generates an oscillating magnetic field
2. an oscillating magnetic field generates an oscillating electric field

Those two points are key to understanding electromagnetic waves.

An electromagnetic wave (such as a radio wave) propagates outwards from the source (an antenna, perhaps) at the speed of light. What this means in practice is that the source has created oscillating electric and magnetic fields, perpendicular to each other, that travel away from the source. The E and B fields, along with being perpendicular to each other, are perpendicular to the direction the wave travels, meaning that an electromagnetic wave is a transverse wave. The energy of the wave is stored in the electric and magnetic fields.

Properties of electromagnetic waves

Something interesting about light, and electromagnetic waves in general, is that no medium is required for the wave to travel through. Other waves, such as sound waves, can not travel through a vacuum. An electromagnetic wave is perfectly happy to do that.

An electromagnetic wave, although it carries no mass, does carry energy. It also has momentum, and can exert pressure (known as radiation pressure). The reason tails of comets point away from the Sun is the radiation pressure exerted on the tail by the light (and other forms of radiation) from the Sun.

The energy carried by an electromagnetic wave is proportional to the frequency of the wave. The wavelength and frequency of the wave are connected via the speed of light:

$c = f \lambda$

Electromagnetic waves are split into different categories based on their frequency (or, equivalently, on their wavelength). In other words, we split up the electromagnetic spectrum based on frequency. Visible light, for example, ranges from violet to red. Violet light has a wavelength of 400 nm, and a frequency of 7.5×10^{14} Hz. Red light has a wavelength of 700 nm, and a frequency of 4.3×10^{14} Hz. Any electromagnetic wave with a frequency (or wavelength) between those extremes can be seen by humans.

Visible light makes up a very small part of the full electromagnetic spectrum. Electromagnetic waves that are of higher energy than visible light (higher frequency, shorter wavelength) include ultraviolet light, X-rays, and gamma rays. Lower energy waves (lower frequency, longer wavelength) include infrared light, microwaves, and radio and television waves.

Energy in an electromagnetic wave

The energy in an electromagnetic wave is tied up in the electric and magnetic fields. In general, the energy per unit volume in an electric field is given by:

$$\text{energy density in an electric field} = 1/2\, \varepsilon_0 E^2$$

In a magnetic field, the energy per unit volume is:

$$\text{energy density in a magnetic field} = 1/2\, B^2 / \mu_0$$

An electromagnetic wave has both electric and magnetic fields, so the total energy density associated with an electromagnetic wave is:

$$u = 1/2\, \varepsilon_0 E^2 + 1/2\, B^2 / \mu_0$$

It turns out that for an electromagnetic wave, the energy associated with the electric field is equal to the energy associated with the magnetic field, so the energy density can be written in terms of just one or the other:

$$u = \varepsilon_0 E^2 = B^2 / \mu_0$$

This also implies that in an electromagnetic wave, E = cB.

A more common way to handle the energy is to look at how much energy is carried by the wave from one place to another. A good measure of this is the intensity of the wave, which is the power that passes perpendicularly through an area divided by the area. The intensity, S, and the energy density are related by a factor of c:

$$S = cu = 1/2\, c\, \varepsilon_0 E^2 + 1/2\, c\, B^2 / \mu_0 = c\, \varepsilon_0 E^2 = c\, B^2 / \mu_0$$

Generally, it's most useful to use the average power, or average intensity, of the wave. To find the average values, you have to use some average for the electric field E and the magnetic field B. The root mean square averages are used; the relationship between the peak and rms values is:

$E_{rms} = E_0 / (2)^{1/2}$ $B_{rms} = B_0 / (2)^{1/2}$

Interference

7-29-99

The wave nature of light

When we discussed the reflection and refraction of light, light was interacting with mirrors and lenses. These objects are much larger than the wavelength of light, so the analysis can be done using geometrical optics, a simple model that uses rays and wave fronts. In this chapter we'll need a more sophisticated model, physical optics, which treats light as a wave. The wave properties of light are important in understanding how light interacts with objects such as narrow openings or thin films that are about the size of the wavelength of light.

Because physical optics deals with light as a wave, it is helpful to have a quick review of waves. The principle of linear superposition is particularly important.

Linear superposition

When two or more waves come together, they will interfere with each other. This interference may be constructive or destructive. If you take two waves and bring them together, they will add wherever a peak from one matches a peak from the other. That's constructive interference. Wherever a peak from one wave matches a trough in another wave, however, they will cancel each other out (or partially cancel, if the amplitudes are different); that's destructive interference.

The most interesting cases of interference usually involve identical waves, with the same amplitude and wavelength, coming together. Consider the case of just two waves, although we can generalize to more than two. If these two waves come from the same source, or from sources that are emitting waves in phase, then the waves will interfere constructively at a certain point if the distance traveled by one wave is the same as, or differs by an integral number of wavelengths from, the path length traveled by the second wave. For the waves to interfere destructively, the path lengths must differ by an integral number of wavelengths plus half a wavelength.

Condition for constructive interference : path length difference = $m\lambda$ $(m = 0, 1, 2, 3, ...)$

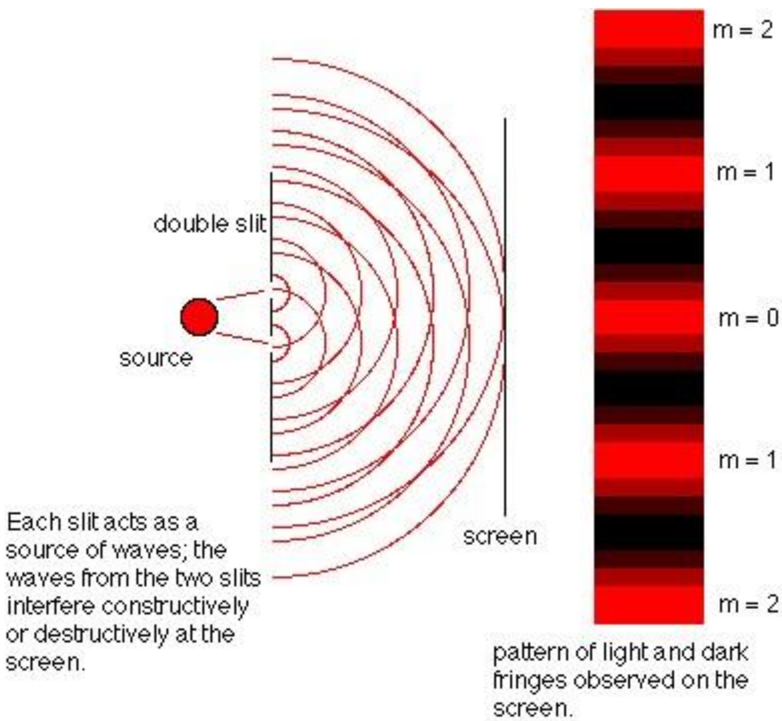
Condition for destructive interference : path length difference = $(m + 1/2)\,\lambda$

Young's double slit

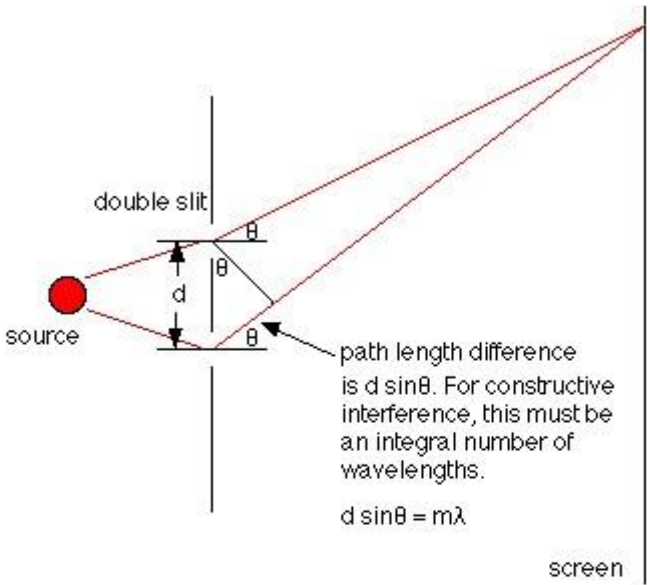
Light, because of its wave properties, will show constructive and destructive interference. This was first shown in 1801 by Thomas Young, who sent sunlight through two narrow slits and showed that an interference pattern could be seen on a screen placed behind the two slits. The interference pattern was a set of alternating bright and dark lines, corresponding to where the light from one slit was alternately constructively and destructively interfering with the light from the second slit.

You might think it would be easier to simply set up two light sources and look at their interference pattern, but the phase relationship between the waves is critically important, and two sources tend to have a randomly varying phase relationship. With a single source shining on two slits, the relative phase of the light emitted from the two slits is kept constant.

This makes use of Huygen's principle, the idea that each point on a wave can be considered to be a source of secondary waves. Applying this to the two slits, each slit acts as a source of light of the same wavelength, with the light from the two slits interfering constructively or destructively to produce an interference pattern of bright and dark lines.



This pattern of bright and dark lines is known as a fringe pattern, and is easy to see on a screen. The bright fringe in the middle is caused by light from the two slits traveling the same distance to the screen; this is known as the zero-order fringe. The dark fringes on either side of the zero-order fringe are caused by light from one slit traveling half a wavelength further than light from the other slit. These are followed by the first-order fringes (one on each side of the zero-order fringe), caused by light from one slit traveling a wavelength further than light from the other slit, and so on.



The diagram above shows the geometry for the fringe pattern. For two slits separated by a distance d, and emitting light at a particular wavelength, light will constructively interfere at certain angles. These angles are found by applying the condition for constructive interference, which in this case becomes:

bright fringes of a double slit: $d \sin\theta = m \lambda$ $(m = 0, 1, 2, 3, \ldots)$

The angles at which dark fringes occur can be found be applying the condition for destructive interference:

dark fringes of a double slit : $d \sin\theta = (m + 1/2) \lambda$ $(m = 0, 1, 2, 3, \ldots)$

If the interference pattern was being viewed on a screen a distance L from the slits, the wavelength can be found from the equation:

$$\lambda = y d / (mL)$$

where y is the distance from the center of the interference pattern to the mth bright line in the pattern. That applies as long as the angle is small (i.e., y must be small compared to L).

Dispersion

Although we talk about an index of refraction for a particular material, that is really an average value. The index of refraction actually depends on the frequency of light (or, equivalently, the wavelength). For visible light, light of different colors means light of different wavelength. Red light has a wavelength of about 700 nm, while violet, at the other end of the visible spectrum, has a wavelength of about 400 nm.

This doesn't mean that all violet light is at 400 nm. There are different shades of violet, so violet light actually covers a range of wavelengths near 400 nm. Likewise, all the different shades of red light cover a range near 700 nm.

Because the refractive index depends on the wavelength, light of different colors (i.e., wavelengths) travels at different speeds in a particular material, so they will be refracted through slightly different angles inside the material. This is called dispersion, because light is dispersed into colors by the material.

When you see a rainbow in the sky, you're seeing something produced by dispersion and internal reflection of light in water droplets in the atmosphere. Light from the sun enters a spherical raindrop, and the different colors are refracted at different angles, reflected off the back of the drop, and then bent again when they emerge from the drop. The different colors, which were all combined in white light, are now dispersed and travel in slightly different directions. You see red light coming from water droplets higher in the sky than violet light. The other colors are found between these, making a rainbow.

Rainbows are usually seen as half circles. If you were in a plane or on a very tall building or mountain, however, you could see a complete circle. In double rainbows the second, dimmer, band, which is higher in the sky than the first, comes from light reflected twice inside a raindrop. This reverses the order of the colors in the second band.

The quantum mechanical view of the atom

8-10-99

Heisenberg uncertainty principle

The uncertainty principle is a rather interesting idea, stating that it is not possible to measure both the position and momentum of a particle with infinite precision. It also states that the more accurately you measure a particle's position, the less accurately you're able to measure it's momentum, and vice versa.

This idea is really not relevant when you're making measurements of large objects. It is relevant, however, when you're looking at very small objects such as electrons. Consider that you're trying to measure the position of an electron. To do so, you bounce photons off the electron; by figuring out the time it takes for each photon to come back to you, you can figure out where the electron is. The more photons you use, the more precisely you can measure the electron's position.

However, each time a photon bounces off the electron, momentum is transferred to the electron. The more photons you use, the more momentum is transferred, and because you can't measure that momentum transferred to infinite precision the more uncertainty you're introducing in the measurement of the momentum of the electron.

Heisenberg showed that there is a limit to the accuracy you can measure things:

Heisenberg uncertainty principle : $\Delta y \, \Delta p \geq h / 2\pi$
where Δy is the uncertainty in the measured position, and Δp is the uncertainty in the momentum.

The uncertainty can also be stated in terms of the energy of a particle in a particular state, and the time in which the particle is in that state:

Heisenberg uncertainty principle : $\Delta E \, \Delta t \geq h / 2\pi$
where Δt is the time interval during which the particle is in a state with energy E.

Quantum numbers

The Bohr model of the atom involves a single quantum number, the integer n that appears in the expression for the energy of an electron in an orbit. This picture of electrons orbiting a nucleus in well-defined orbits, the way planets orbit the Sun, is not our modern view of the atom. We now picture the nucleus surrounded by electron clouds, so the orbitals are not at all well-defined; we still find the Bohr theory to be useful, however, because it gives the right answer for the energy of the electron orbitals.

The Bohr model uses one quantum number, but a full quantum mechanical treatment requires four quantum numbers to characterize the electron orbitals. These are known as the principal quantum number, the orbital quantum number, the magnetic quantum number, and the spin quantum number. These are all associated with particular physical properties.

n, the principal quantum number, is associated with the total energy, the same way it is in the Bohr model. In fact, calculating the energy from the quantum mechanical wave function gives the expression Bohr derived for the energy:

Energy : $E_n = -(13.6 \text{ eV}) \; Z^2 / n^2$ $\qquad$ $(n = 1, 2, 3, ...)$

ℓ, the orbital quantum number, is connected to the total angular momentum of the electron. This quantum number is an integer less than n, and the total angular momentum of the electron can be calculated using:

Total angular momentum : $L = [\, \ell(\ell+1) \,]^{1/2} \, h / 2\pi$ $\qquad$ $(\ell = 0, 1, 2, ..., n-1)$

m_ℓ, the magnetic quantum number, is related to one particular component of the angular momentum. By convention, we call this the z-component. The energy of any orbital depends on the magnetic quantum number only when the atom is in an external magnetic field. This quantum number is also an integer; it can be positive or negative, but it has a magnitude less than or equal to the orbital quantum number. The z-component of the electron's angular momentum is given by:

$L_z = m_\ell h / 2\pi \qquad (m_\ell = -\ell, ..., -2, -1, 0, 1, 2, ..., \ell)$

m_s, the spin quantum number is related to something called the spin angular momentum of the electron. The closest analogy is that it's similar to the Earth spinning on its axis. There are only two possible states for this quantum number, often referred to as spin up and spin down.

$m_s = +1/2$ or $m_s = -1/2$

What's the use of having all these quantum numbers? We need all four to completely describe the state an electron occupies in the atom.

Electron probability density clouds

A very important difference between the Bohr model and the full quantum mechanical treatment of the atom is that Bohr proposed that the electrons were found in very welldefined circular orbits around the nucleus, while the quantum mechanical picture of the atom has the electron essentially spread out into a cloud. We call this a probability density cloud, because the density of the cloud tells us what the probability is of finding the electron at a particular distance from the nucleus.

In quantum mechanics, something called a wave function is associated with each electron state in an atom. The probability of finding an electron at a particular distance from the nucleus is related to the square of the wave function, so these electron probability density clouds are basically three-dimensional pictures of the square of the wave function.

The Pauli exclusion principle

If you've got a hydrogen atom, with only a single electron, it's very easy to determine the possible states that electron can occupy. A particular state means one particular combination of the 4 quantum numbers; there are an infinite number of states available, but the electron is more likely to occupy a low-energy state (i.e., a low n state) than a higher-energy (higher n) state.

What happens for other elements, when there is more than one electron to worry about? Can all the electrons be found in one state, the ground state, for example? It turns out that this is forbidden: the Pauli exclusion principle states that no two electrons can occupy the same state. In other words, no two electrons can have the same set of 4 quantum numbers.

Shells and subshells

As usual, for historical reasons we have more than one way to characterize an electron state in an atom. We can do it using the 4 quantum numbers, or we can use the notion of shells and subshells. A shell consists of all those states with the same value of n, the principal quantum number. A subshell groups all the states within one shell with the same value of ℓ, the orbital quantum number.

The subshells are usually referred to by letters, rather than by the corresponding value of the orbital quantum number. The letters s, p, d, f, g, and h stand for values of 0, 1, 2, 3, 4, and 5, respectively. Using these letters allows us to use a shorthand to denote how many electrons are in a subshell; this is useful for specifying the ground state (lowest energy state) of a particular atom.

The ground state configuration for oxygen, for instance, can be written as :

oxygen : $1s^2\ 2s^2\ 2p^4$

This means that the lowest energy configuration of oxygen, with 8 electrons, is to have two electrons in the n=1 s-subshell, two in the n=2 s-subshell, and four in the n=2 psubshell.

Potassium (Z = 19) has an interesting ground state configuration:

potassium : $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^1$

That's interesting because there is a d-subshell in the n = 3 shell, but instead of the last electron going into that subshell it goes into the s-subshell of the n=4 shell. It does this to minimize the energy: the 4s subshell is at a lower energy than the 3d subshell.

The periodic table

When Mendeleev organized the elements into the periodic table, he knew nothing about quantum numbers and subshells. The way the elements are organized in the periodic table, however, is directly related to how the electrons fill the levels in the different shells.

Different columns of the periodic table group elements with similar properties; they have similar properties because of the similarities between their ground state electron configurations. The noble gases (He, Ne, Ar, etc.) are all in the right-most column of the periodic table. Their ground state configurations have no partially filled subshells; having a complete subshell is favorable from the standpoint of minimizing energy so these elements do not react readily.

On the other hand, the column next to the noble gases is the halogens; these are one electron short of having completely-filled subshells, so if they can share an electron from another element they're happy to do so. They react readily with elements whose ground state configurations have a single electron in one subshell like the alkali metals (Li, Na, K, etc.).

ELECTROMAGNETIC INDUCTION without a MAGNETIC FIELD by

Christian MONSTEIN, CH-8807 FREIENBACH/Switzerland

With a simple experiment the following will be shown: The presence alone of a variable magnetic field, with respectto time and location in an electric conductor, is not a necessary prerequisite for the induction of an electric voltage. The generally considered flux rule to be the correct one, according to MAXWELL-FARADAY, is not valid for the experiment described.

1. INTRODUCTION

In generator systems the LORENTZ force is frequently used to explain the driving force for the current flux. (Named after Hendrik Antoon LORENTZ, 1853-1928). The expert has to grapple with the Left-Hand-Rule [1], so that formula (1) can be applied correctly since the vectors F, v and B are all orthogonal (at right angles) to each other.

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad (1)$$

The empirically derived induction law by FARADAY states, that if a magnetic flux around a path becomes time variable then a voltage is induced in it proportional to the magnetic flux change. In order to calculate the induced voltage, one has to first calculate the magnetic flux (Gr. phi) from the flux density **B**.

$$\phi = \int \mathbf{B} \cdot d\mathbf{A} \qquad (2)$$

The time variation of the flux means

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathbf{A} \qquad (3)$$

The induced voltage in a coil can also be described as a line integral thus

$$U_{ind} = \oint \mathbf{E} \cdot d\mathbf{s} \qquad (4)$$

Considering LENZ's Law we derive the induced voltage (electromotive force or source voltage). This variation of the induction voltage is also called the source voltage of rest (steady state) [2].

$$U_{ind} = -N \cdot \frac{\partial \phi}{\partial t} \quad (5)$$

where N = number of turns in the coil frame surface **A.**

Analogous to this there is a source voltage of motion, an alternate derivation based on the law of induction.

$$U_{ind} = -Z \cdot L \cdot (\mathbf{B} \times \mathbf{v}) \quad (6)$$

where Z = the number of conductors, L = length of conductor at right angle to **B** and **v**, v = velocity of conductor motion.

Equations (5) and (6) are usually applied in engineering and seem most often to work. But they only show two special cases of a much more generalized situation. This general situation has been deliberately achieved in the HOOPER-MONSTEIN-Experiment[3],[4],[5],[6],[7]. See also Fig. 1 for this purpose (taken from[7].

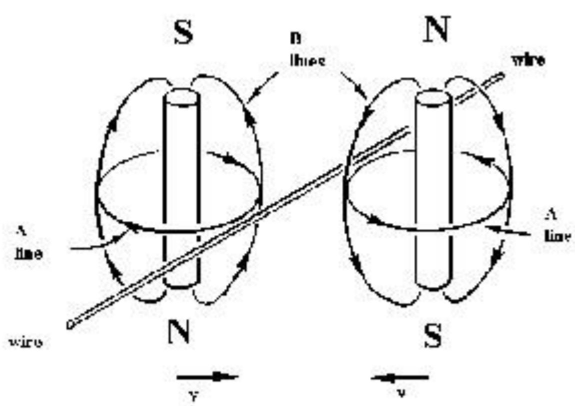
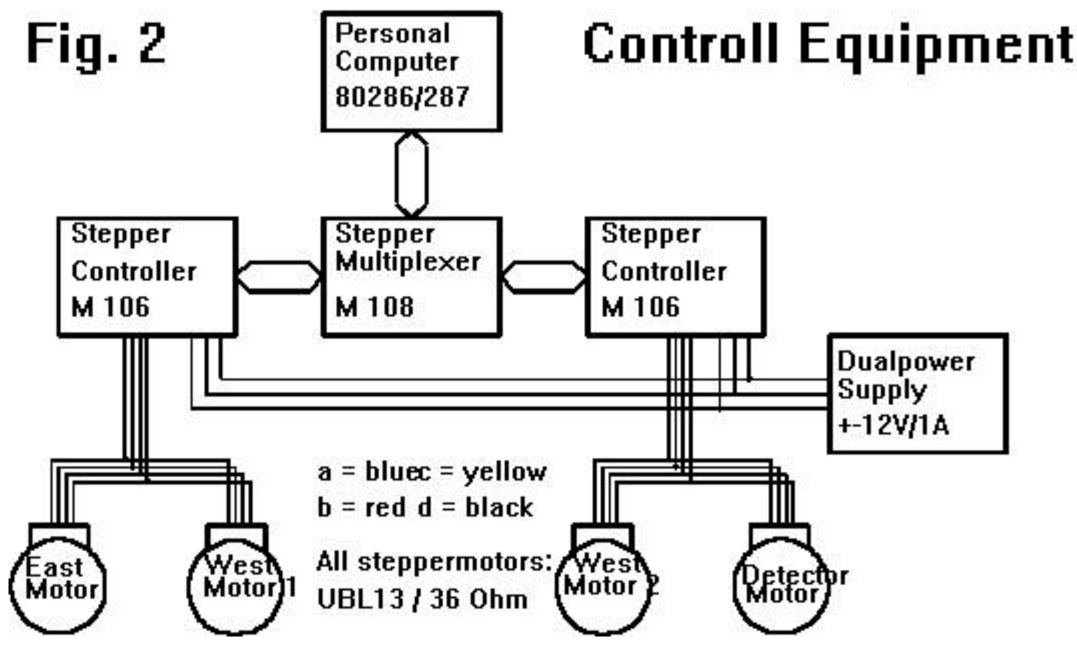


Fig. 1

---

2. EXPERIMENT

Two identical, permanent magnets have both been placed orthogonally (at a right angle) to a pilot or test wire which has an active length of 150 mm. The magnetic pole area density is about 50 mT. The magnets are so placed that unequal polarities are opposite each other. This placement causes the magnetic field to be equal to zero at a point in the center where the pilot wire is located. The two representative field lines (B lines) drawn near the pilot wire, show opposite arrows to indicate the extinction of the magnetic field. This can easily be checked with a HALL-probe. The two magnets, as well as the test wire (pilot wire) are each distinctly movable by means of a stepper motor in a direction 'v' as indicated. The control is automatically handled from a PC (personal computer) with external hardware for 4 stepper motors. See Fig. 2, Control Equipment, below. The entire complicated mechanical setup with sled, motors, bearings, test frame etc. was again supplied and loaned to me free by my friend Mr. Hanspeter BENZ. My thanks to him! My contribution was limited to the control-PC and the control software, as well as the measuring circuit and the evaluation.



Range of Motion

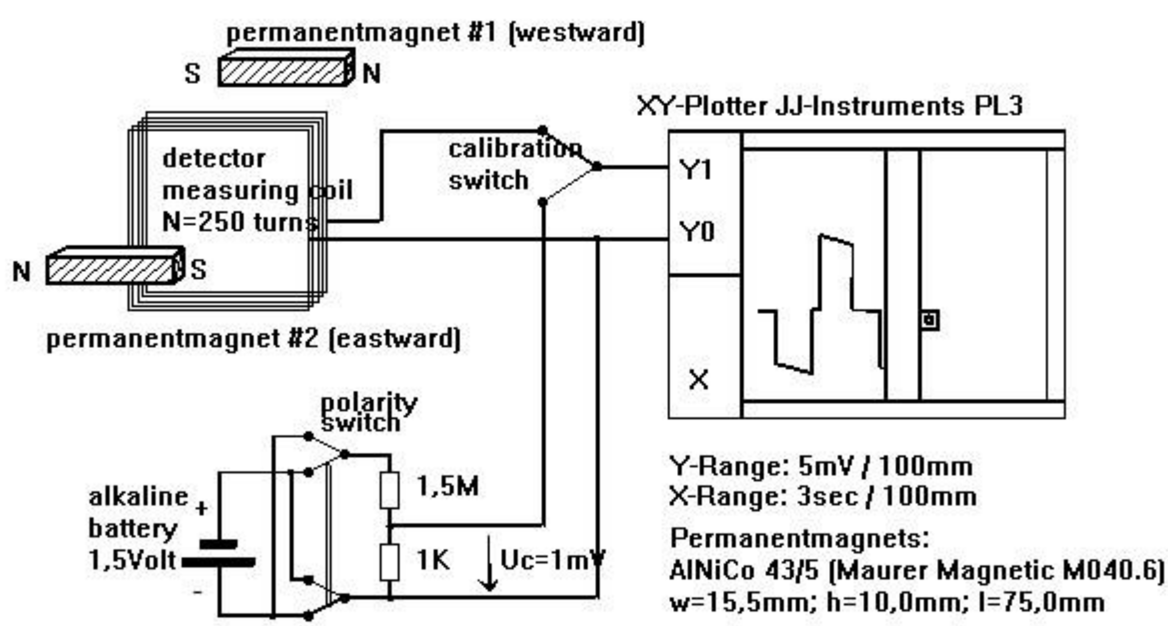
Using this convenient possibility to move the magnets as well as the test wire at will, allows a multitude of variations and combinations of motion. Most interesting, above all, are those movements in the center area, where the pilot wire is located, and where the magnetic flux density is and remains at zero. For all the following experiments 'v' is kept constant at 2.64 mm/s, so that parts of the experiment can be compared. The selected velocity is dependent on the maximum timing frequency for the stepper motors derived from the PC, on the one hand and the possible range of motion (Gr. delta) 's' on the other. The range of motion (Gr. delta) 's' results from the maximum number of position steps np = 240 multiplied by the amount of one position step ë (Gr. delta) 's'. A single position step 's' = 0.022 mm is dependent on the spindle drive and the number of active windings of the linear stepper motors. The maximum possible measurement distance, respectively range of motion (Gr. delta) 's' = np x 's', which computes to about 7.29 mm. This range of motion 's' is carried out, depending on the particular part of the experiment, either by the 'left' magnet, and/or the 'right' magnet and/or the pilot wire. Some of the variations carried out are listed in the table shown in Fig. 4.

Measuring Instruments

The actual measuring instrument used during this experiment is a sufficiently sensitive XY(t) plotter with a measurement range of 5 æV per 100 mm y-deflection. The x-axis is handled with an internal deflection generator with a range of 3 seconds per 100mm xdeflection. The pilot wire can be electrically disconnected and be replaced with a calibrated circuit. This calibration circuit is driven with a battery and a resistive voltage divider producing a constant voltage of about 1 mV.

The wiring diagram for the measuring circuit is shown below in Fig. 3. Additionally it is possible to invert the voltage polarity of the calibration circuit. In order to increase the sensitivity we used a wired frame, instead of a single wire, with n = 250 turns. However, the values given in the Table shown in Fig. 4 have been recalculated for a single pilot wire. The entire experiment is subdivided into three partial experiments as described below.

Fig. 3 Measuring Circuit



Moved (East) Magnet

In the first experiment only one of the two magnets is moved towards the pilot wire. The other magnet as well as the pilot wire are stationary. This is the 'normal' case for which the induction law, as it is known, is approximately valid, since in this case **B** <> 0. When the magnet approaches the pilot wire, **B** does increase linearly. Along the pilot wire the average magnetic flux density B is about 0.8 mT, which was measured with a HALL-probe KSY10. This allows to estimate the expected induced voltage in the test wire

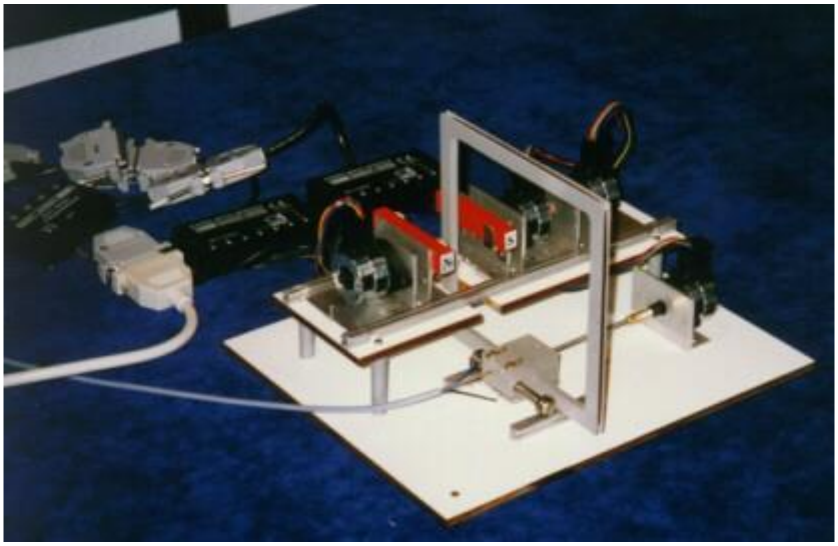
according to the conventional method which is u = BLv = 0.8 mT x 15 cm x 2.64 mm/s = 0.3µ V. This of course is much too small a value since essential portions of the magnetic flux of the two magnets are already cancelling. The values measured were 1.2µV - 1.6µV moving from East to West and 1.3µV - 1.8µV when moving from West to
East.

Both Magnets Moved

In the second epxeriment both magnets were simultaneously moved toward the test wire and after a short delay retracted from the pilot wire. Both magnets were moving at the same velocity **v** and covered the same distance (Gr. Delta) s. In the center the resulting magnetic field is exactly zero. This means that according to Equations (5) and (6) no voltage should be induced, since B = 0 mT as well as (Gr. phi) = 0 Vs are, at all times and at all locations of the pilot wire equal to zero. The fact, however, is that in this case double the voltage of the first experiment, namely 2.4µV - 3.6µV were measured. This is a crass violation of the induction law which had been considered correct for decades! This experiment nicely shows that the induction laws, as everyone knows them, can not be correct. It also shows that it is not the magnetic flux **B** which is responsible for the induced voltage. It is something else, namely the vector potential **A**, which, as shown in above situation, is not cancelled out to zero, but is instead doubled. (See the arrows for the A-line in Fig.1.)

Moved (West) Magnet and Test Wire

In many physics books it is stated that the magnetic flux **B** is real and the magnetic vector potential **A** is a purely mathematical magnitude. Based on these experiments we have to invert that statement and say, rather that **B** is a pure fiction and **A** is real. (whatever it may mean that something is physically real). Another interesting variation is the third (partial) experiment. Here the left (East) magnet remains stationary. The right (West) magnet is instead moved with the double velocity from the right towards the pilot wire. At the same time the pilot wire itself is moved to the left with the velocity **v** towards the stationary magnet. This assures that the distances between the test wire and the magnets remain symmetrical. Related to this again, is a cancellation of the magnetic flux at all times at the location of the test wire. Although again in this case the induction formulas (5) and (6) fail, just as in experiment 2, double the voltage is reached namely 2.6µV - 3.6µV . In this case also the relativity of motions is fulfilled, i.e. for the end result it does not matter whether the test wire or the magnets are moved!



Picture of the experimental installation

---

3. RESULT

All voltages given are average values derived from several experiments. The measurement uncertainty for all partial measurements is ± 0,12µV. Figur 4: Table

Para-meter	Velocity towards center; v = 2,64mm/sec ± 0,01mm/sec			Magnetic Flux density	Voltage ± 0,12µV at Test wire L=15cm	
Expe-riment #	Magnet left (East)	Test wire center	Magnet right (West)	Permanent-magnets	begin at x_0	end at x_0+Ds
0	0	0	0	no magnets	0,0µV=0	0,0µV=0
1a	v	0	0	B <> 0	1,2µV=U	1,6µV=U'
1b	0	0	v	B <> 0	1,3µV=U	1,8µV=U'
2	v	0	v	B=0	2,4µV=2U	3,6µV=2U'
3	0	v	2v	B=0	2,6µV=2U	3,6µV=2U'

In the two columns on the far right of the TABLE in Fig. 4 each time there are given two voltage values U and U', which refer to the beginning (U) and the end (U') of the range of motion (gr. Delta) 's'. Since **A** depends on the location (it decreases radially outward), there results an induction voltage that is also dependent on location (at constant linear velocity). Beyond the partial experiments listed above, ten other variations were conducted, however, they are not listed here, since there was no change in the end result. It is and remains a fact that **A** rather than **B** is the cause for the induced voltage.

4. INDUCTION THROUGH CHANGE OF THE MAGNETIC VECTOR POTENTIAL

A corrected formula for calculating the induced voltage must not start with the magnetic flux **B**, but rather with the magnetic vector potential **A**, and it must also consider the changes of location and those depending on the time. The situation with time changes of **A** lead to an electrical field strength which by different authors [5] is called "motionaltransformer electric intensity".

$$\mathbf{E}_{mtr} = \frac{-\partial \mathbf{A}}{\partial t} \quad (7)$$

From this we can determine the induced voltage by using a line integral around the conductor path. If, as in our example, for instance the velocity of the magnets **v** is linear, then (7) can be rewritten as follows

$$\mathbf{E}_{mtr} = -\left(\mathbf{v} \cdot grad\right) \cdot \mathbf{A} \quad \text{oder} \quad -\left(\mathbf{v} \cdot \nabla\right) \cdot \mathbf{A} \quad (8)$$

For a case in which the magnets are stationary and the conductor is moving, a different starting situation results for the induced field strength. In order to distinguish this field strength in[5] it is called "motional electric intensity".

$$\mathbf{E}_{mot} = \mathbf{v} \times rot\mathbf{A} \quad \text{oder} \quad \mathbf{v} \times \left(\nabla \times \mathbf{A}\right) \quad (9)$$

[Please note: rot = curl]

Therefore, summed up, the "global" electric field strength is:

$$\mathbf{E}_{global} = \mathbf{E}_{mot} + \mathbf{E}_{mtr} = \mathbf{v} \times \left(\nabla \times \mathbf{A}\right) - \left(\mathbf{v} \cdot \nabla\right) \cdot \mathbf{A} \quad (10)$$

Now therefore the actually induced voltage in the measuring circuit can be calculated

$$U_{ind} = N \cdot \oint \left[\mathbf{v} \times \left(\nabla \times \mathbf{A}\right) - \left(\mathbf{v} \cdot \nabla\right) \cdot \mathbf{A}\right] \cdot d\mathbf{s} \quad (11)$$

where N = number of turns

LITERATURE

[1] Hans Breuer, dtv-Atlas zur Physik Band 2 1991, Elektrizität, Magnetismus, Festkörper, Moderne Physik, ISBN 3-423-03227-8 [Atlas for physics volume 2, 1991, Electricity, Magnetism, Solid State Bodies, Modern Physics]

[2] Georg Rose, Grosse Elektronik-Formelsammlung für Radio-/Fernsehpraktiker und Elektroniker. Franzis-Verlag Poing 1995. ISBN 3-7723-5339-8 (in German)

[3] W. J. Hooper, PhD, New Horizons In Electric, Magnetic, And Gravitational Field Theory, Electrodynamic Gravity Inc. 1974 (available trough Tesla Book Company)

[4] Oliver Crane, Zentraler Oszillator und Raum-Quanten-Medium, Band 1, section 3, pages 203..211, Universal Experten Verlag Rapperswil 1992. ISBN 3-9520261-0-7 (In German only).

[5] Christian Monstein, Deutsche Physik (German Physics) 1(4) 1992, Electromagnetic Induction Without Magnetic Field.

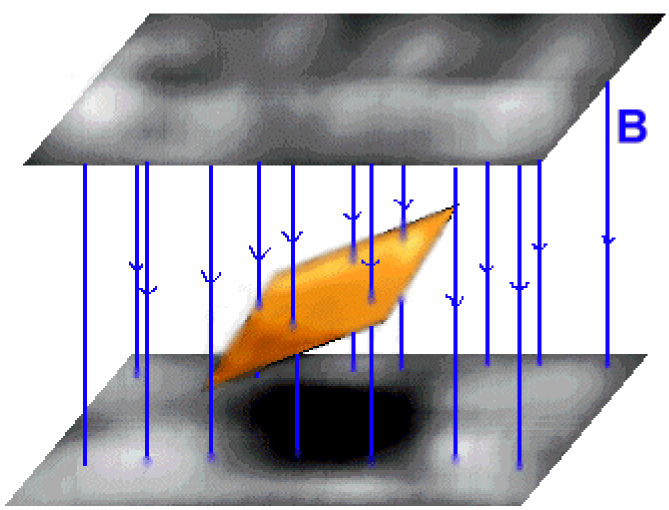
[6] J. P. Wesley, Deutsche Physik 2(5) 1993, Correspondence to the editor of Deutsche Physik.

[7] J. P. Wesley, Classical quantum Theory, pages 314...316, Benjamin Wesley Blumberg 1996.

[8] R. P. Feynman, R. B. Leighton, M. L. Sands, The Feynman lectures on physics, Band II, Teil 1, Bilingua - Ausg. R Oldenbourgh und Addison - Wesley, Aufl. 1973 - 1977, ISBN 3-486-33701-7.

Introduction to induction:

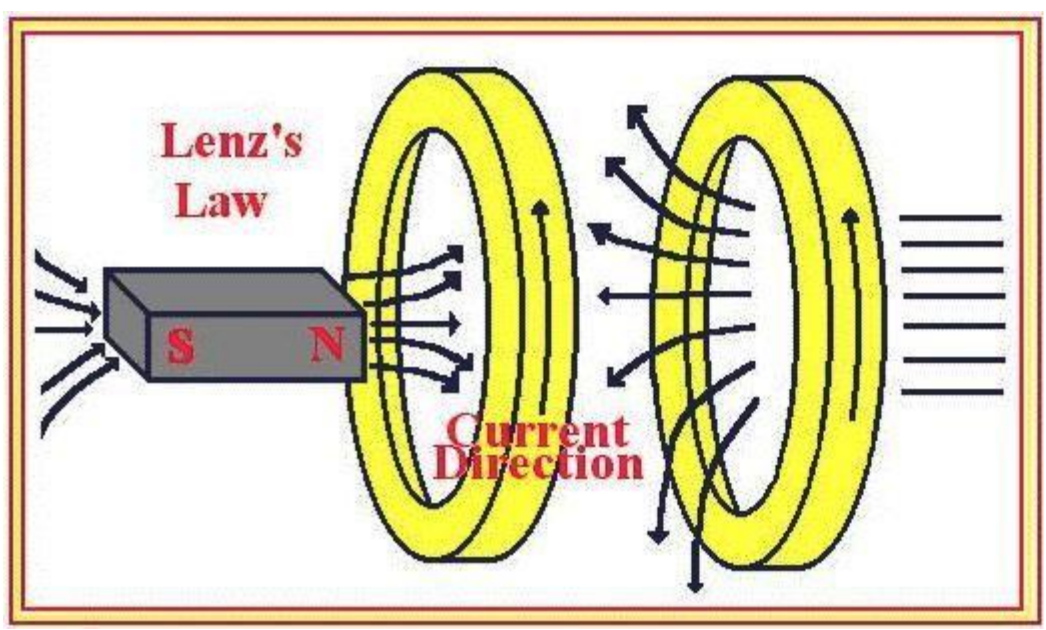
Magnetic flux is a term for the density of a magnetic field. Flux is measured by a unit called a *weber* (wb), which is a tesla x a square meter. Flux is proportional to the number of magnetic field lines passing through a surface.



Flux is the number of magnetic field lines passing through a surface. As you can see in the diagram, this depends on both the strength of the magnetic field (stronger field = more lines = larger flux) and the size of the surface (larger surface = more lines = larger flux).

When the flux through a surface *changes*, an electromotive force (emf - and not really a vector force as we know it) is *induced*. This ***induced emf*** causes ***induced current***.

As long as the magnetic field is changing, a current is induced. The induced current is always in such a direction as to *keep the magnetic field constant*. The emf induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit. Basically this means that the induced emf is directly related to magnetic flux as a function of time.



Lenz's Law states that the polarity of an induced emf will produce a current that creates a magnetic flux that opposes the change in magnetic flux through the loop formed by the circuit. The induced current opposes any change in magnetic flux.

That is to say, the induced current must be in a direction so that the flux it produces opposes the change in the external (applied) magnetic flux. For example, if a bar moves to the right, then the induced current would be counterclockwise so that the force produced is directed towards the left.

There. A brief overview of flux and induced current. Move on to Project, Part A

Well, the answer is the Simpsons, physics, and biology.

Sort of.

More specifically:

Suppose Bart Simpson is walking through a constant magnetic field. The flux through his body remains constant, and no current is induced. His neurons are able to maintain their relatively negative internal charge (see Rest.html), and when a stimulus causes them to be moved to an action potential, potassium ions can diffuse in and cause the inside of the neuron to temporarily be more positively charged than the outside (see Action.html). In fact, Bart's entire nervous system is made up of neurons that function in this way.

However, if Bart were to stroll through a *changing* magnetic field, some current would be induced in his body. The current would most likely be induced along pathways in his body which carry electricity already, such as those of the nervous system. If the change in flux induced current that caused every single positively charged potassium ion to leave Bart's neurons, the neurons would sense something very wrong with their situation, and would undergo a process called apoptosis.

Apoptosis is, essentially, cell suicide by means of a few of its own specialized enzymes. This would be a horrible fate because, as most people know already, nervous system cells do not reproduce. Bart would have forever lost those neurons. Neuronal apoptosis could cause him to gradually lose control of his body.

However, if Bart were very old, and were to stroll through a changing magnetic field which induced current such that the outward flow of potassium ions from his neurons was reduced, it could slow down the rate of their death and he could avoid Alzheimer's disease.

Interesting, no?

For the complete technical run-down, see http://www.alzh98.com/book/p1270.htm which, surprisingly enough, is completely understandable after a year of both AP Physics and AP Biology.

Now that we are so enlightened, let's move on to some physics application questions.

Question: Does Flux really have something to do with calculus?

Hope: God, no.

Answer: Why yes, yes it does.

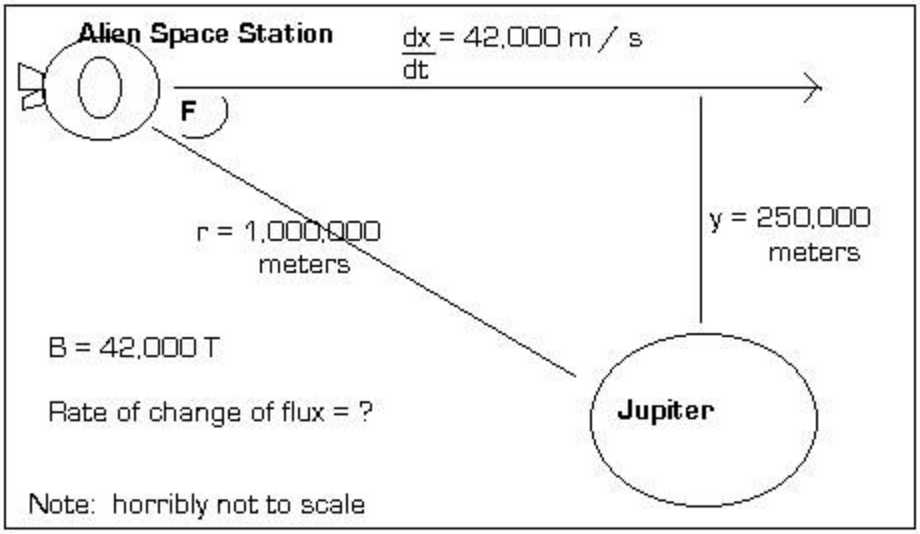
In fact, here's a flux problem involving calculus and related rates. Let's try it, shall we?

Problem:

A circular space station with area 4,200 square meters containing many highranking alien military personnel is propelling itself past Jupiter at 42,000 m/s. They are traveling on a line tangent to an orbit 250,000 meters from Jupiter and the ring of their space station is parallel to that tangent (see highly artistic diagram below). At the instant when the station is a million meters from Jupiter, at what rate is the flux through the station changing?

(Assume Jupiter's magnetic field to be a constant 420 Teslas. Although they radiate outward from Jupiter, assume the magnetic field lines to be essentially parallel by the time they get to the station. Also, disregard any effects that have no bearing on the problem.)

Diagram:



Solution:

Let the above angle F be the angle made by the station and the magnetic field lines.

Taking the derivative of Flux = BA cos F, we find that d(flux)/dt = (-BA sin F)(dF/dt)

We need only to find dF/dt.

Using the Pythagorean Theorem, x^2 + y^2 = r^2
Therefore, x^2 = (1,000,000)^2 - (250,000)^2 >>> x = 968,245 m

Taking the derivative of the Pythagorean Theorem,
2x dx/dt + 2y dy/dt = 2r dr/dt >>> 2(968,245)(42,000) + 2(250,000)(0) = 2(1,000,000) dr/dt, solving which gives dr/dt = 40666 m/s (effectively negative)

We know that cos F = x/r by definition of cosine, so we can take the derivative

$$-\sin F \frac{df}{dt} = \frac{r\frac{dx}{dt} + x\frac{dr}{dt}}{r^2}$$

and get:

Because d(flux)/dt = BA(-sin F df/dt), we can substitute and

$$\frac{d\,(\text{flux})}{dt} = BA\left(\frac{r\frac{dx}{dt} + x\frac{dr}{dt}}{r^2}\right)$$

get:

We can then substitute directly and

$$\frac{d\,(\text{flux})}{dt} = (42{,}000)(4{,}200)\left(\frac{(1{,}000{,}000)(42{,}000) + (968{,}245)(-40666)}{(1{,}000{,}000)^2}\right)$$

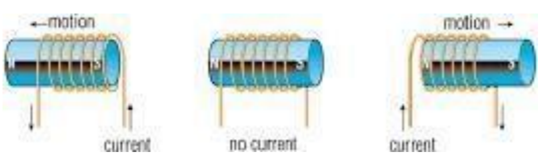
get:

Which simplifies to:

d(flux)/dt = 463,111 Wb / S

That's a lot. That's *some* current. Let's hope the aliens thought ahead.

Phew. Head to my final sort of wrap-up page, the musings or
go back to the contents



electromagnetic induction

Movement of a magnet in a coil of wire induces a current. (Image Â© Helicon)

In electronics, the production of an electromotive force (emf) in a circuit by a change of magnetic flux through the circuit or by relative motion of the circuit and the magnetic flux. As a magnet is moved in and out of a coil of wire in a closed circuit an induced current will be produced. All dynamos and generators produce electricity using this effect. When magnetic tape is driven past the playback head (a small coil) of a tape recorder, the moving magnetic field induces an emf in the head, which is then amplified to reproduce the recorded sounds.

Electromagnetic induction takes place when the magnetic field around a conductor changes. If the magnetic field is made to change quickly, the size of the current induced is larger. A galvanometer can be used to measure the direction of the current. As a magnet is pushed into a coil, the needle on the galvanometer moves in one direction. As the magnet is removed from the coil, the needle moves in the opposite direction.

If the change of magnetic flux is due to a variation in the current flowing in the same circuit, the phenomenon is known as self-induction if it is due to a change of current flowing in another circuit it is known as mutual induction.

Lenz's law
The direction of an electromagnetically induced current (generated by moving a magnet near a wire or a wire in a magnetic field) will be such as to oppose the motion producing it. This law is named after the German physicist Heinrich Friedrich Lenz (18041865), who announced it in 1833.

Faraday's laws
English scientist Michael Faraday proposed three laws of electromagnetic induction (1) a changing magnetic field induces an electromagnetic force in a conductor (2) the electromagnetic force is proportional to the rate of change of the field (3) the direction of the induced electromagnetic force depends on the orientation of the field.

Magnetic Fields and Magnetic Forces

Properties of magnets:

1. A magnet has polarity - it has a north and a south pole; you cannot isolate the north or the south pole (there is no magnetic monopole)
2. Like poles repel; unlike poles attract
3. A compass is a suspended magnet (its north pole is attracted to a magnetic south pole); the earth's magnetic south pole is within 200 miles of the earth's geographic north pole (that is why a compass points "north")
4. Some metals can be turned in to temporary magnets by bringing them close to a magnet; magnetism is induced by aligning areas called domains within a magnetic field
5. Permanent magnets are formed of metallic alloys or metals such as iron, nickel, or cobat

Magnetic field (symbol is *B* and SI unit is the Tesla or *T*) the environment around a magnet in which the magnetic forces act. Another common unit for magnetic field strength is the *gauss (G)*; 1 G = 1 x 10^{-4}

Magnetic field lines they represent the area around a magnet; magnetic field lines outside of the magnet flow from the north to the south pole

Domain

Atoms of ferromagnetic materials act in groups called domains; atomic magnets in each domain are aligned so that each domain is a microscopic bar magnet; the domains align themselves with an external magnetic field. Each domain behaves like a tiny magnet and has a north and a south pole. In unmagnetized materials, the domains are randomly arranged. In magnetized materials, the domains are aligned. Anything that randomizes the alignment of the domains destroys the magnetic properties of a material (dropping a magnet or heating it) **Comparing electricity and magnetism:**

Electricity	**Magnetism**
+ and - charges	N and S poles
like charges repel	like poles repel

unlike charges attract	unlike poles attract
electric monopole exists	no magnetic monopole
electric field lines flow from + to -	magnetic field lines flow from N to S
density of lines equals strength of E	density of lines equals strength of B
SI unit: ampere, 1 A = 1 C/sec	SI unit: Tesla, 1 T = 1 N/Amp meter
E exerts force on a charge, or E = F/q	Field exerts force on a moving charge, or B = F/(qvsin□)

Curie temperature temperature above which a material loses all magnetic properties

Oersted (1820) found that an electric current in a wire produces a magnetic field around it; a stationary charge does not create a magnetic field

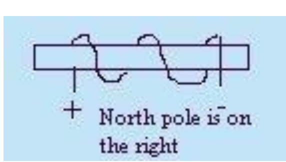
Right-hand rules predict the direction of magnetic fields produced by a current. They are used for conventional current flow. Use your left hand to predict the direction an electron or negative charge would follow.

RHR #1 - Straight Wire Conductor

Curl the fingers of the right hand into the shape of a circle. Point the thumb in the direction of the current and the tips of the fingers will point in the direction of the magnetic field.

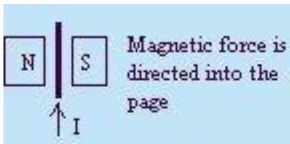
RHR #2 - Solenoid

Curl the fingers of the right hand in the direction of the current. Your thumb is the north pole of the electromagnet.



RHR #3 - Magnetic Force

Extend the right hand so that the fingers point in the direction of the magnetic field and the thumb points in the direction of the current. The palm of the hand then pushes in the direction of the magnetic force.



Forces Due to Magnetic Fields

Ampere found that a force is exerted on a current-carrying wire in a magnetic field

F = B I L sin □ where *B* is the magnetic field in Teslas (T), *I* is the curent, *L* is the length of wire in meters, and □ is the angle. Only the perpendicular component of *B* exerts a force on the wire. If the direction of the current is perpendicular to the field
(□=90), then the force is given by
F = B I L

We know how to measure force, current, and length. Thus *B* can be calculated by using

$$B = \frac{F}{I\,L}$$

The force produced by a magnetic field on a single charge depends upon the speed of the charge, the strength of the field, and the magnitude of the charge.

F = q v B sin □
where *q* is the charge in Coulombs and *v* is the velocity of the charge. If □=90, then F = q v B

How speed affects the force on a charged particle moving in a magnetic field.
Effects of speed of particle in a Magnetic Field

If the charged particle moves parallel to the field lines (□=0), then the magnetic force on the particle is zero. If a charged particle is moving perpendicular to a uniform magnetic field, the path of the charged particle is an arc (or circle). The magnetic force is the source of the centripetal force on the charged particle. This relationship can be used to find the radius of the arc. $(m v^2)/ r = q v B$

Since the magnetic force is perpendicular to the velocity of the charged particle, the force does not cause the speed of the particle to change, only its direction. Thus, no work is done by the magnetic force on the charged particle.

Deflection of electron in a Magnetic Field due to Magnetic Force

The magnetic field near a long straight wire is directly proportional to the current *I* in the wire and inversely proportional to the distance *r* from the wire. The magnetic field at any point a distance *R* away from a straight-wire conductor can be calculated using,

$$B = \frac{(2 \times 10^{-7}) I}{R}$$

or, it can be written in its true form (**This is an important formula for the AP B exam.**)

$$B = \frac{\mu_o I}{2 \pi r}$$

where μ_o is a constant called the permeability of free space and has a value of $4\pi \times 10^{-7}$ T m/A

Since a wire carrying a current produces a magnetic field and the wire experiences a force when placed in a magnetic field, two current-carrying wires exert a force on each other. The force exerted on the second wire is only due to the magnetic field exerted by the first wire. Parallel currents in the same directions attract each other and parallel currents in opposite directions repel each other.

Force on a Loop of Wire (represented by multiple choice questions on the AP B exam in which you predict the direction of current, etc.) At the center of the loop, the magnetic field is perpendicular to the plane of the loop. If there are *N* loops, the strength of the magnetic field at the center of the loop is given by multiplying the following by *N*. The direction of the magnetic field at the center of the loop can be determined using a RHR (the thumb is pointed in the direction of the current and the curled fingers are placed at the center of the loop, then the palm pushes in the direction of the magnetic field.)

$$B = \frac{\mu_o I}{2 r}$$

Electromagnetic Induction

There are two ways that electricity and magnetism are related: an electric current produces a magnetic field and a magnetic field exerts a force on an electric current or moving charged particle. Henry and Faraday independently found found that a current could be induced in a wire by moving it in a magnetic field. An electric current is generated in a wire when the wire cuts across magnetic field lines.

Faraday found that a steady magnetic field does not produce any current, only a *changing* magnetic field produces an electric current.

Hints for the AP B exam:

1. Approximately, two out of every three years one of the free response questions involves a situation where a charge is accelerated through two charged plates. The charge then enters a magnetic field whose direction causes the charge to move in a circle. These are common questions that are asked:
    - Calculate the speed of the charge as it exits the region between the two charged plates.
    - Draw the direction of the electric field between the two charged plates.
    - If there is also a magnetic field between the two charged plates in addition to the electric field, explain the relationship between the two fields that allows the charge to pass through undeflected.
    - Calculate the radius of the path of the charge in the magnetic field.
2. Remember these three formulas for regions where both electric and magnetic fields exist: $V=Ed$, $qE=F$, and $F=qvB$. Manipulating these formulas allows you to express the velocity of the charge in terms of E and B. Manipulating these formulas allow you to write an expression for the accelerating voltage in terms of v, B, and d.
3. Remember, if the charge is moving in a circle, the magnetic force provides the centripetal force. This allows you to calculate the radius.
4. Remember, if the charge is moving in a circle and the magnetic field is perpendicular to it, it does no work on the charge. It only changes its direction.
5. Sometimes they ask you to calculate the thermal energy dissipated by the accelerated charge if it is allowed to strike a target. You know its speed; calculate its energy.
6. A mass spectrometer is also representative of this type of problem. In a mass spectrometer, the radius of the path of a particle is proportional to its mass. If you have several particles, you can set up a proportion between their masses and radii to determine the mass of an unknown particle.

7. If the charged particle moves in a circular path, the centripetal force equals the magnetic force. This equality can be solved for the ratio of charge to mass of the particle (q/m).
8. More points are awarded when they ask for a direction if you express it in terms of positive or negative x, y, or z.

Electromagnetic induction process of generating a current by using a magnetic field. This is sometimes called motional emf. emf = B L v sin □ where *emf* is the potential difference measured in volts, *v* is the velocity with which the wire is moved through the magnetic field *B*, □ is the angle at which the wire is moved in the magnetic field, and *L* is the length of the wire **electromotive force (emf)** a potential difference, measured in volts, that can cause an induced current to flow in a wire. It is not a force, but is a historical term coined before electricity was understood.
An induced emf is produced by a changing magnetic field.

Electromagnetic Induction

the process where current is produced when either a wire or a magnetic field move relative to one another; as long as the wire cuts across magnetic field lines during the motion, a current is produced. A current is induced in a coil of wire if it is moved into or out of a magnetic field; a current is induced in a coil of wire if a magnet is inserted or removed from the coil of wire. It doesn't matter if the magnet or the coil moves-motion or change is required to induce an emf.

Lenz's Law

The direction of the induced current is such that the magnetic field resulting from the induced current opposes the change in the flow (or flux) that causes the induced current. It is the *change* in the flow or flux that causes the induced current, not the flux itself.

How I predict the direction of the induced current using Lenz's Law:

1. Determine whether the magnetic field strength is increasing or decreasing.
2. Determine the direction in which the original field enters the coil.
3. Determine the direction of the induced magnetic field so that it opposes the change in the magnetic flux.
4. Use RHR to predict the direction of the current knowing the direction of the induced magnetic field.

AP B Multiple Choice Questions Hints: There are always questions asked in which you much predict the direction of an induced current (or emf, □)

1. Know the situations when a current or emf is induced in a coil or wire (Look under the explanation for Faraday's law). Remember - there must be relative motion or something that causes a changing flux!
2. Be able to write an expression (or calculate) for induced current or emf. In other words, this can be easily done using a combination of □ = Blv sin □ and □ = iR.
3. KNOW your RHR that enables you to predict the current through a single loop or coil. Remember, your thumb points in the direction of the current, clockwise or counterclockwise. Your fingers enter the loop or coil in the direction of the magnetic field.
4. Be able to interpret directions in terms of the x, y, and z axes.
5. Be able to calculate (or write an expression) induced voltage using Faraday's Law. Faraday's law can also be used to calculate the rate of change of the magnetic field in a moving coil (in other words, it can calcualte B/t).
6. The faster something is moved, the greater the induced voltage (current)

   because emf is directly proportional to the velocity. **Self-inductance**

   induced *emf* produced in a coil by a changing current **Mutual inductance**

   a changing current in one coil induces an *emf* in another coil

Transformer

an electrical device that increases or decreases AC voltage; a step-up transformer has more turns in the secondary than in the primary; a step-down transformer has more turns in the primary than in the secondary. We will call the primary the incoming voltage or current and the secondary the outgoing voltage or current.

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \text{or} \quad \frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$$

where *N* is number of turns, *V* is the voltage, and *I* is the current. *s* and *p* stand for secondary and primary, respectively.

Magnetic flux (□$_B$ and SI unit is the Weber, Wb)the number of magnetic field lines that pass through a surface of area *A*. A changing magnetic flux produces an electric field. This is true not only of wires and conductors, but also applies to any region in space.

□$_B$ = $B_{perpendicular}$ A where $B_{perpendicular}$ is the component of B perpendicular to the face of the coil

Faraday's Law of Induction Faraday found that the amount of emf induced in a coil of wire depended upon how rapidly the magnetic field changes in the coil of wire. The faster the magnetic field changes, the greater the induced emf. If the flux through a coil of *N* loops of wire chagnes by an amount $\square_B$ during a time $\square$t.

$\square = - N (\square_B/\square t)$

The negative sign indicates the direction in which the induced emf acts. For our purposes, we will use Faraday's law to calculate the magnitude of the induced emf and apply right hand rules for Lenz's Law to determine the direction of the induced emf.

An emf can be induced three ways:

1. By a changing magnetic field
2. By changing the area of the loop in the field
3. By changing the loop's orientation with respect to the field

Electric Motors and Generators

Electric motor uses electrical energy to produce mechanical energy. In a motor, there must be a source of a magnetic field; *brushes* serve as a connection to the *split-ring commutator*, allowing current flow from the motor to an outside source. In order to continue rotating, current direction must be reversed. This is achieved by the use of the split-ring commutator and the brushes. The force on a current-carrying wire in a magnetic field causes an electric motor to rotate

The Electric Motor

Electric generator uses mechanical energy to create electrical energy; rotation of wire loop in a

magnetic field causes current to be induced. This current changes direction every 180 degrees, producing *alternating current* (AC current).

The Electric Generator

Magnetic Moment When an electric current flows in a closed loop of wire placed in a magnetic field, the magnetic force on the current can cause a torque. This is the basic principle behind meters and motors. If the coil consists of *N* loops of wire carrying current *I* with area *A*, the torque is given by

$\square$ = NIAB sin $\square$ where NIA is the vector quantity called the magnetic dipole moment of the coil. Its direction is perpendicular to the plane of the coil.

Magnetism Sample Problems

Magnetism Homework

AP Magnetostatics Hand Rule Class Problems

AP Magnetostatics Sample Problems

AP Electromagnetic Induction Sample Problems

AP Lenz's Law Class Problems

AP Magnetostatics Objectives

AP Electromagnetism Objectives